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Nonextensivity and galaxy clustering in the Universe

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Abstract

We investigate two important questions about the use of the nonextensive thermostatistics (NETS) formalism in the context of nonlinear galaxy clustering in the Universe. Firstly, we define a quantitative criterion for justifying nonextensivity at different physical scales. Then, we discuss the physics behind the ansatz of the entropic parameter $q(r)$. Our results suggest the approximate range where nonextensivity can be justified and, hence, give some support to the applicability of NETS to the study of large-scale structures.

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1. Introduction

The evolution of large-scale structures in the Universe is one of the most important questions of modern cosmology. A considerable amount of work has been done on numerical and analytical approaches to characterise the clustering of matter at large scales. One of the difficulties inherent to this subject is that, in order to make progress in the understanding of general models, it is necessary to define methods for structure quantification.

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Usually, quantitative studies of large-scale structures are based on the two-point correlation function $\xi(r)$ for the galaxy distribution. Estimates indicate this function is well approximated by $\xi(r) = (r/r_0)^{-\gamma}$ (with $\gamma \approx 1.8$), where r_0 is the correlation length, the scale marking the transition between linear and nonlinear regimes. The usual range for γ, found in the literature, is $1.5 < \gamma < 1.97$ (see, e.g., Refs. [1,2]), with r_0=5 h^{-1} Mpc for galaxies and r_0=25 h^{-1} Mpc for cluster (see, e.g., Ref. [3]). At larger scales, the structure of the Universe presents patterns like walls and filaments (with dimensions $\sim$ 150 h^{-1} Mpc) seemingly reaching homogenisation at Cosmic Microwave Background (CMB) scales ($\gtrsim$ 1000 h^{-1} Mpc). Some authors have had some success describing the clustering properties of visible matter over this wide range of scales in terms of a multifractal phenomenon associated with density thresholds applied to multifractal sets (e.g. Refs. [4,5]). However, the relative success of the multifractal approach does not imply a better understanding of the physics behind this framework. Actually, it is not simple to find a dynamical connection between fractal sets and galaxy clustering. Recently, Ramos et al. [6] (hereafter RWRR) shed some light on this question by putting forward a model based on the generalized thermodynamics (NETS) formalism [7]. They show that applying the idea of nonextensivity, intrinsic to NETS, it is possible to derive an expression for the correlation function

$$1 + \xi(r) = \tfrac{1}{3} D_2 r^{(D_2-3)} , \tag{1}$$

using a scale-dependent correlation dimension

$$D_2(r) = 3\, \frac{\log[2 + a(1 - q(r))]}{\log 2} , \tag{2}$$

where the entropic parameter $q(r)$ is given by the following ansatz:

$$r \sim \frac{1}{(q-1)^{\beta}} \tag{3}$$

with a and β being free parameters of the model. This approach shows a smooth transition from a clustered Universe to large-scale homogeneity, with $D_2 = 3$. However, RWRR do not discuss two important questions concerning the conceptual basis of the model: the necessity of defining a criterion that allows us to assume nonextensivity in the context of galaxy clustering and the physics behind the ansatz for $q(r)$. In the next two sections these questions are further developed and some conclusions are drawn at the end.

2. Nonextensivity and gravitational clustering

The physical motivation of the approach adopted by RWRR is built upon the fact that components of gravitating systems tend to evolve spontaneously into increasingly complex structures due to the long-range nature of the gravitational interaction. The NETS theory generalises the Boltzmann–Gibbs statistical mechanics and can be applied to systems dominated by the long-range nature of gravity. However, the application of NETS to an ensemble of comoving cells containing gravitating particles depends on the behaviour of the average correlation energy inside a spherical cell of volume

$V = 4\pi R^3/3$ with increasing scales R. For instance, the grand canonical ensemble of cells that are larger than the correlation length are approximately extensive, since the universal expansion effectively limits the thermodynamic effects of gravity to roughly the correlation length scale (e.g. Refs. [3,8]). Also, for an individual cell whose size is larger than the correlation length, extensivity is possibly a good approximation because the correlation energy between two members of the ensemble is negligible compared to the internal correlation energy:

$$U_{corr} \ll U_1 + U_2 \Rightarrow U_{tot} \approx U_1 + U_2 . \tag{4}$$

Due to these reasons, the application of NETS to galaxy clustering is not straightforward and demands better criteria to properly describe the problem using the nonextensive formalism. For real nonextensive systems, the correlation energy between two cells should be as important as the internal correlation energy, such that

$$U_{tot} = U_1 + U_2 + U_{corr} . \tag{5}$$

Following Sheth and Saslaw [9], we compute the average gravitational correlation energy within cells of volumes V and $2V$ for increasing R:

$$W_V = \bar{n}V \int_0^R \frac{Gm^2}{2r}\, \xi(r) 4\pi\bar{n}r^2\, \mathrm{d}r , \tag{6}$$

where m is the mass and $\bar{n}$ is the average number density of particles in a cell of size V. The extensivity approximation requires that $W_{2V} \approx 2W_V$, approximately verified for the power law correlation, $\xi(r) = \xi_0 r^{-\gamma}$, whenever $\gamma \geqslant 1$ (see Ref. [9]). In this case, $|W_{2V}/2W_V| = 2^{(2-\gamma)/3}$, which means that, using $\gamma = 1.77$, we find a $\sim 5\%$ deviation from the strict extensivity condition at all scales.

In the NETS context, we substitute (1), (2) and (3) into (6), numerically integrate for V and $2V$ (the spherical cell of volume $2V$ has radius $2^{1/3}R$) and calculate the correlation ratio $|W_{2V}/2W_V|$ for each upper limit R. The results are presented in Fig. 1, where we plot the three cases investigated by RWRR in comparison to the line defined by the power-law correlation (for $\gamma = 1.77$). Note that, for the three NETS models, we see different levels of deviations from the extensivity approximation. In particular, NETS_3 model presents deviations from extensivity of $\sim 20\%$ even at very large scales. However, for large enough scales, the correlation ratio decreases and the extensivity approximation is recovered, for models NETS_1 and NETS_2. In order to properly invoke nonextensivity, we propose a lower limit for the ratio $|W_{2V}/2W_V|$ as 10%. This is twice the energy ratio for the power-law correlation case.

One can further quantify this as follows. Consider two elliptical galaxies of radii r_g separated by the distance r_0. A family of analytic models for spheroidal stellar systems is defined by the density distribution:

$$\rho_\eta(r) = \frac{\eta}{4\pi} \frac{1}{r^{3-\eta}(1+r)^{1+\eta}} \tag{7}$$

for $0 < \eta \leqslant 3$ [10]. Choosing units in which the total mass and the gravitational constant are both unity, we find the self-gravitational energy of each galaxy as

$$U_1 = U_2 = -\frac{1}{2}\left(\frac{1}{2\eta - 1}\right) = U_\eta , \tag{8}$$

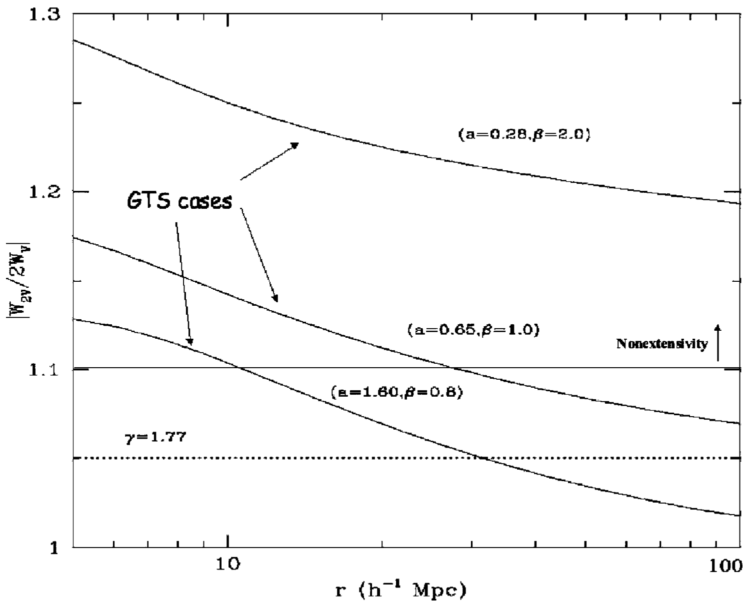

Fig. 1. Behaviour of the correlation energy ratio with the scale R.

and the potential at a distance r_0 is

$$\Phi_\eta(r_0) = \frac{1}{\eta - 1}\left[\frac{r_0^{\eta-1}}{(1+r_0)^{\eta-1}} - 1\right] , \tag{9}$$

with the mass interior to radius r_g being

$$M_\eta(r_g) = \frac{r_g^\eta}{(1+r_g)^\eta} . \tag{10}$$

The gravitational energy of the interacting system formed by the two galaxies is given by

$$U_{int} = 2\Phi_\eta(r_0)M_\eta(r_g) . \tag{11}$$

Assuming we have only the two galaxies in the volume defined by $4\pi r_0^3/3$, we should have $U_{corr} = U_{int}$. Then $U_{tot} = 2U_\eta + U_{int}$ and, consequently,

$$\frac{U_{tot}}{2U_\eta} = 1 - 2\left(\frac{2\eta - 1}{\eta - 1}\right)\left[\frac{r_0^{\eta-1}}{(1+r_0)^{\eta-1}} - 1\right]\left[\frac{r_g^\eta}{(1+r_g)^\eta}\right] . \tag{12}$$

In Fig. 2, we present the behaviour of $U_{tot}/2U_\eta$ as a function of r_0 (for $r_0 < 5$ h^{-1} Mpc and taking $r_g = 30$ h^{-1} kpc). Note that only at very small scales the ratio significantly increases. Actually, at the galaxy correlation length, the deviation from extensivity is about 1%. As a comparison, NETS models give $\sim$ 13%, 17% and 28%, for the cases 1, 2 and 3, respectively, at the same scale, in the more general situation. Thus, a NETS model with at least 10% of deviation from the strict extensivity condition corresponds

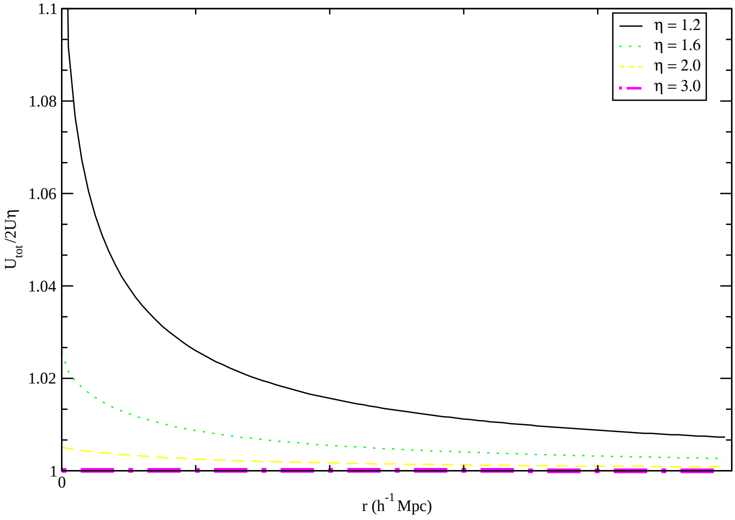

Fig. 2. Correlation energy ratio for η models. For comparison, the Hernquist's model is computed for $\eta = 2$.

to 10 times the deviation in the case of a cell with only two typical elliptical galaxies interacting within the galaxy correlation length ($r_0 = 5\ \mathrm{h}^{-1}$).

Using this criterion, NETS_1 and NETS_2 models are well justified for $r < 10\ \mathrm{h}^{-1}$ Mpc and $r < 30\ \mathrm{h}^{-1}$ Mpc, respectively, while NETS_3 model is valid at all scales. This result reinforces the idea of applying the NETS formalism to galaxy clustering phenomena, but clearly indicating the approximate range it will be used, given a specific choice of the free parameters a and β. Hence, the level of nonextensivity strongly depends on the ansatz (3), which reinforces the need of a deeper discussion of the meaning of $q(r)$.

3. The physics behind $q(r)$

Structure formation models try to compute the evolution of cosmic structure from the very early Universe to the present day. Usually, they are stochastic, in the sense that random initial conditions are used, with well-specified statistical properties, and their late evolution may be highly non-linear [11]. Any attempt to compute $q(r)$ from first principles must take into account this complex scenario.

Considering the relevant role played by turbulence in the dynamics of structure formation (e.g. Refs. [12,13]), a natural approach for deriving a meaningful expression for $q(r)$ is to assume a *a la* Kolmogorov cascade phenomenology, in which the energy supplied at large scales by the physical mechanisms acting over structure formation flows down until being finally dissipated on the smallest scales by viscous processes [14]. The key ingredient in this "top-down" cascade scenario is the presumption of the existence, within a certain range of scales, of a scaling $\langle v_r^n \rangle \sim r^{\zeta_n}$ of the moments of the peculiar velocity differences $v_r(x) = v(x+r) - v(x)$. This scale-invariance can be rigorously deduced, under assumptions such as local isotropy, from the dynamical equations governing a turbulent fluid [15]. In the present cosmological context, scale-invariance is well supported by observations [16] and is at the heart of the fractal description of galaxy clustering [4]. If we now assume that the probability density function (PDF) of peculiar velocity differences is described by the Tsallis canonical distribution and that at sufficiently large scales turbulent fluctuations are normally distributed, then an analytical expression for $q(r)$ can be easily derived [17], in the form of $q \sim (15 - 21r^{\alpha})/(9 - 15r^{\alpha})$, where $\alpha = \zeta_4 - 2\zeta_2$ and ζ_p are the structure function exponents.

This model was successfully applied to hydrodynamics turbulence ([18,19]), where the entropic parameter represents a direct measure of intermittency. It gives similar results to those obtained with Eq. (3), albeit only within a restricted range of scales. Note that, as $r \rightarrow 0$, the entropic parameter tends to a finite value, which is physically expected for any model of $q(r)$ [20]. But in this case, Eq. (3) would not provide a smooth transition from small-scale fractality to large-scale homogeneity, which represents a major drawback in the present cosmological context.

The problem with such a cascade phenomenology is its description of structure formation from larger to smaller scales. To support observations, a different and more compelling model may be obtained from a "bottom-up" fractal cascading scenario. In this cold dark matter (CDM) dominated Universe scenario, large-scale structures are formed by gravitational clustering of smaller clumps of matter [20]. This process eventually leads to strong density fluctuations with self similar density distribution and a stationary fractal dimension, while enhancing long-range correlations and the corresponding high-energy tails in the peculiar velocity differences in PDFs.

In this context, averaging a Gaussian conditional velocity distribution over all possible spatio-temporal energy dissipation rate fluctuations and at appropriate scales leads to a Tsallis peculiar velocity differences in PDF and to a closed-form expression for the entropic parameter [21]. This expression depends on the (discrete) number of degrees of freedom relevant to represent the local fluctuations. It can be further generalised, taking the form $q \sim (r+3)/(r+1)$. Considering the lack of observational data for an unambiguous determination of $q(r)$, we foresee a complementary (and computationally expensive) approach to gain a deeper insight on the role of the entropic parameter. We propose to compute estimates of q within a large range of scales directly from N-body simulations, using a Λ-CDM cosmological model of gravitational clustering.

4. Summary and conclusions

The basis for clustering statistics in cosmology is the study of galaxy distribution in the Universe. The analysis of an increasing amount of astronomical data can provide the correct framework to explain the formation and evolution of large-scale structure in the Universe. In particular, correlation functions suggest the possibility of describing the galaxy clustering properties in the context of the multifractal approach. Recently, RWRR presented a better physical interpretation to this approach by deriving multifractality from the NETS formalism. Now, extending their work, we define a quantitative criterion to use the NETS formalism based on deviations from strict extensivity. Assuming it should reach at least 10 times the expected deviation level within a cell containing only two typical galaxies, we obtain the approximate validity domain for the NETS models: $r < 10$ Mpc for NETS_1 model, $r < 30$ Mpc for NETS_2, and at any scale for NETS_3. The choice of $q(r)$ has a physical meaning in the context of hydrodynamics turbulence and can be understood as a "bottom-up" fractal cascading, in conceptual agreement with CDM structure formation models. Although the scenario we propose is not complete, we should keep in mind that there is not a unified framework to explain galaxy clustering in the Universe. Further numerical developments of NETS models will be presented in a forthcoming paper [22].

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