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Multiscaling and nonextensivity of large-scale structures in the Universe

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Abstract

There has been a trend in the past decade to describe the large-scale structures in the Universe as a (multi)fractal set. However, one of the main objections raised by the opponents of this approach deals with the transition to homogeneity. Moreover, they claim there is not enough sampling space to determine a scaling index which characterizes a (multi)fractal set. In this work we propose an alternative solution to this problem, using the generalized thermostatistics formalism. We show that applying the idea of nonextensivity, intrinsic to this approach, it is possible to derive an expression for the correlation function, describing the scaling properties of large-scale structures in the Universe and the transition to homogeneity, which is in good agreement with observational data.

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1. Introduction

One of the key problems in structure formation theories is the issue of the galaxy distribution and the transition to homogeneity and isotropy of the Universe in large enough scales. Historically, the hypothesis of homogeneity (the cosmological principle, CP) was introduced by Einstein to find simple solutions of the field equations for the case where the spatial hypersphere of the Universe is a maximally symmetric subspace of the space–time. That allows us to derive the Robertson–Walker metric and the Friedmann equations, the theoretical framework where cosmology has been developed. At the same time, the CP implies that

all mass units should be statistically equivalent, with a Poisson distribution in space. In this case, correlations may appear only on average and should be the same when viewed from any point in the system. Actually, it has been shown that luminous matter in the Universe shows a quite structured distribution of galaxies and voids. This structure has led some authors [1] to tentatively describe galaxy clustering using a fractal distribution with a dimension ~ 1.2 . The results of a number of redshift surveys [2,3] indicate that, indeed, there is a certain hierarchy in the Universe, with stars forming galaxies, galaxies grouping themselves in groups, clusters and superclusters. A rough limit to this clustering is seen at a distance of about $200 h^{-1}$ Mpc and the present redshift surveys do not show evidence of large-scale structures beyond this scale. Particularly, in very large scales—those probed by cosmic microwave background (CMB) experiments, espe-

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cially by the COBE-DMR experiment—there is no evidence of violation of local isotropy [4]. However, we should note that the existence of such a crossover towards homogenization, as well as the exact value of the fractal dimension, are questions of intense debate [5–7].

The fractal hypothesis is deeply connected with a topology theorem which states that homogeneity is implied by the condition of local isotropy plus the assumption of analicity (or regularity) for the distribution of matter. It is possible to prove that in a fractal the condition of local isotropy can be satisfied but, since a fractal is a nonanalytic structure, the property of homogeneity is not implied [8]. This means that the fractal scenario is incompatible with the CP. Observations point out that galaxy structures indeed exhibit fractal properties up to some scale, although a pure fractal description does not seem to be favored at the moment [9]. Recently, some authors (e.g. [10–12]) have had some success in describing the clustering properties of visible matter in the Universe in terms of a multifractal phenomenon associated with density thresholds applied to multifractal sets. They show that both a hybrid fractal and a multifractal approaches can reasonably describe the matter distribution up to $\approx 100 h^{-1}$ Mpc.

In spite of the relative success of the multifractal approach to describe the distribution of matter in the Universe, it is important to understand the physics behind this framework. In general terms, the multifractal description of galaxies may represent a strange attractor that is the nonlinear outcome of the dynamical equations of gravitational galaxy clustering [13,14]. However, it is not simple to find a dynamical connection between fractal sets and galaxy clustering. In this work we propose an alternative solution to this problem, using the generalized thermostatics (GTS) formalism [15]. We show that applying the idea of nonextensivity, intrinsic to GTS, it is possible to derive an expression for the correlation function, describing the scaling properties of large-scale structures in the Universe. The present approach is based on the assumption of a scale-dependent correlation dimension (D_2), which leads to a reconciliation with observational data at various scales, showing a

smooth transition from a clustered, fractal Universe to large-scale homogeneity, with $D_2 = 3$.

The physical motivation behind our approach is the peculiar behavior of large-scale gravitational systems, dominated by the unshielded, long-range nature of gravity. In contrast, other many-body systems, like neutral gases and plasmas, are characterized by short-range interactions. Because of this fundamental difference, the standard Boltzmann–Gibbs statistical mechanics cannot be applied to gravitating systems, since the long-range nature of gravity strongly violates one of its basic premises (short-ranged effective interactions), and, thus, special techniques are needed [16]. One possibility to overcome this difficulty—adopted by us—is to use the GTS, a theory proposed to correct Boltzmann–Gibbs statistical mechanics in those cases where its prescriptions fail.

2. Theoretical framework

In the context of galaxy clustering, the important exponent is the correlation dimension D_2 [10], which is defined in terms of the scaling of the correlation integral $C(r)$ over a distance s (normalized by a reference scale L):

$$C(r) = \int_0^r 4\pi n(1 + \xi(s))s^2 ds = Ar^{D_2}, \quad (1)$$

where $\xi(s)$ is the two-point correlation function, n the mean density of objects and A is a constant. By differentiating Eq. (1) with respect to r we get

$$1 + \xi(r) = \frac{D_2 Ar^{(D_2-3)}}{4\pi n}. \quad (2)$$

Considering that in large enough scales we have $(1 + \xi) \rightarrow 1$ and $D_2 \rightarrow 3$, we obtain $A = 4\pi n/3$ and

$$1 + \xi(r) = \frac{1}{3} D_2 r^{(D_2-3)}. \quad (3)$$

Based on the scaling properties of multifractals, a generalization of Boltzmann–Gibbs thermostatics has been proposed [15] through the introduction of a family of generalized entropy functionals S_q with a single parameter q . These functionals reduce to the classical, extensive Boltzmann–Gibbs form as $q \rightarrow 1$. The generalized entropy has been successfully used to describe

a wide range of phenomena, including long-range interactions [17], turbulence [18–20], anomalous diffusion [21], among others. For an up-to-date list of references on GTS and its applications, see [22].

For a system with W microscopic state probabilities $p_i \geq 0$, that are normalized according to $\sum_i^W p_i = 1$, the GTS formalism is based upon two axioms. First, the entropy of the system is given by

$$S_q = k \frac{1 - \sum_i^W p_i^q}{q - 1} = \frac{k}{q - 1} \sum_i^W (p_i - p_i^q),$$

where k and q are real constants. Second, an experimental measurement of an observable O , whose value in state i is o_i , yields the (unnormalized) q -expectation value,

$$O_q = \sum_i^W p_i^q o_i,$$

of the observable O .

Within the framework of GTS, the following special additivity rule holds:

$$O_q(2V) = 2O_q(V) + \frac{2(1 - q)O_q(V)S_q(V)}{k}, \quad (4)$$

where V is an arbitrary volume. In the sense of Eq. (4), the entropic index q characterizes the degree of nonextensivity of the system.

Nonextensivity implies that O_q is not uniformly distributed within V but rather concentrated on a subset of noninteger dimension D_2 . In this case, the following scaling relation holds:

$$O_q(2V) = 2^{D_2/3} O_q(V). \quad (5)$$

The reasoning above may be extended to the context of galaxy clustering if we assume the correlation integral $C(r)$ as being the observable in Eq. (4). Thus, with the additional assumption of a scale-dependent entropic parameter $q(r)$, from Eqs. (4) and (5), it immediately follows:

$$D_2(r) = 3 \frac{\log(2 + a(1 - q(r)))}{\log 2}, \quad (6)$$

where $D_2(r)$ is the correlation dimension and $a = 2S_q/k$ will be determined later.

The idea of relating nonextensivity and multiscaling through a scale-dependent correlation dimension, as given by Eq. (6), has been originally proposed by us in the context of fully developed turbulence [18,19]. Interestingly, Chen and Bak [23] recently suggested a similar relation, derived from a simple reaction–diffusion model of turbulent phenomena, as a new geometric form for describing the distribution of luminous matter in the Universe [24].

To be complete, the above formulation shall contain a model for $q(r)$. We suggest the following simple ansatz:

$$r \sim \frac{1}{(q - 1)^\beta}, \quad (7)$$

or equivalently $(q - 1) \sim (1/r)^{1/\beta}$, with $\beta > 0$. At large scales, as r grows, we have $q \rightarrow 1$ and $D_2 \rightarrow 3$. The appropriate choice of a sets the value of D_2 at a lower reference scale.

3. Results and discussion

We compared our model against observational data from various redshift surveys [2,3]. Results are shown in Figs. 1 and 2, for $a = 0.65$ and $\beta = 1.0$ (solid line); $a = 1.60$ and $\beta = 0.8$ (dashed line); $a = 0.28$ and $\beta = 2.0$ (long dashed line) with $L = 100 \text{ h}^{-1} \text{ Mpc}$ for all cases. For comparison purposes, a purely fractal description ($D_2 = 2$) and a homogeneous scenario ($D_2 = 3$) are also depicted in Fig. 1 (dotted lines). We observe that for $\beta = 1$, which corresponds to the simplest relation between q and r , our results show a good agreement with observational data for both the two-point correlation function and the correlation dimension. Increasing β , we may obtain a better fitting for $1 + \xi(r)$ at the expense of the correlation dimension data. The opposite is true for decreasing values of β .

Eqs. (3) and (6) offer a quantitative description of matter clustering in the Universe, and of the smooth transition from small-scale nonextensive fractal behavior to large-scale extensive homogeneity. From a geometrical point of view, our model shows a Universe displaying a clear hierarchy, with predominance

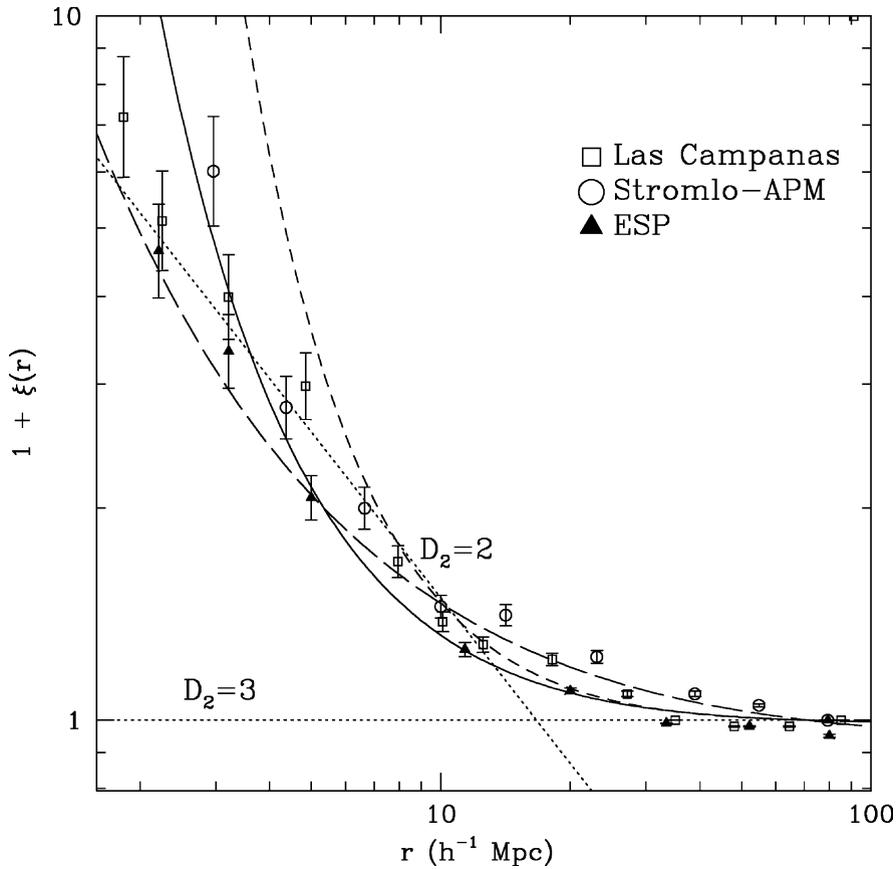


Fig. 1. The two-point correlation function vs. scale, for the Stromlo-APM, the Las Campanas and the ESP redshift surveys [3], and for the present model, for $a = 0.65$ and $\beta = 1.0$ (solid line); $a = 1.60$ and $\beta = 0.8$ (dashed line); $a = 0.28$ and $\beta = 2.0$ (long dashed line) with $L = 100 h^{-1} \text{ Mpc}$ for all cases. For comparison purposes, a purely fractal description ($D_2 = 2$) and a homogeneous scenario ($D_2 = 3$) are also displayed (dotted lines).

of point-like ($D_2 \sim 0$) and filamentary ($D_2 \sim 1$) structures at small scales, and surface-like ones ($D_2 \sim 2$) at intermediate scales. For sufficiently large scales ($r > 500 h^{-1} \text{ Mpc}$), the homogeneity predicted by the CP is recovered. All these features of the present approach are in good agreement with observational data.

Summarizing, we may say that our primary motivation in this work was to investigate the prediction of multiscaling and nonextensivity of large-scale structures in the Universe within the context of the generalized thermostatistics formalism. The results presented above suggest that we cannot discard this theoretical framework as a viable way to explain the gravitational clustering in the Universe. However, we should be

careful and keep in mind two important caveats: the poorness of observational data in some scale domains may be masking the results, and the fact that luminous galaxies may not perfectly trace the mass distribution (the biasing problem). From a theoretical point of view, what is missing at the moment is a detailed understanding of how q vary with scale. In the present work, we adopted the simplest model that provided a good agreement with the data. We are currently running a number of COBE-normalized CDM simulations for some relevant cosmological models. We believe that further study on the resulting correlations and velocity distributions, at different scales (from galaxies to superclusters), may reveal the connection between

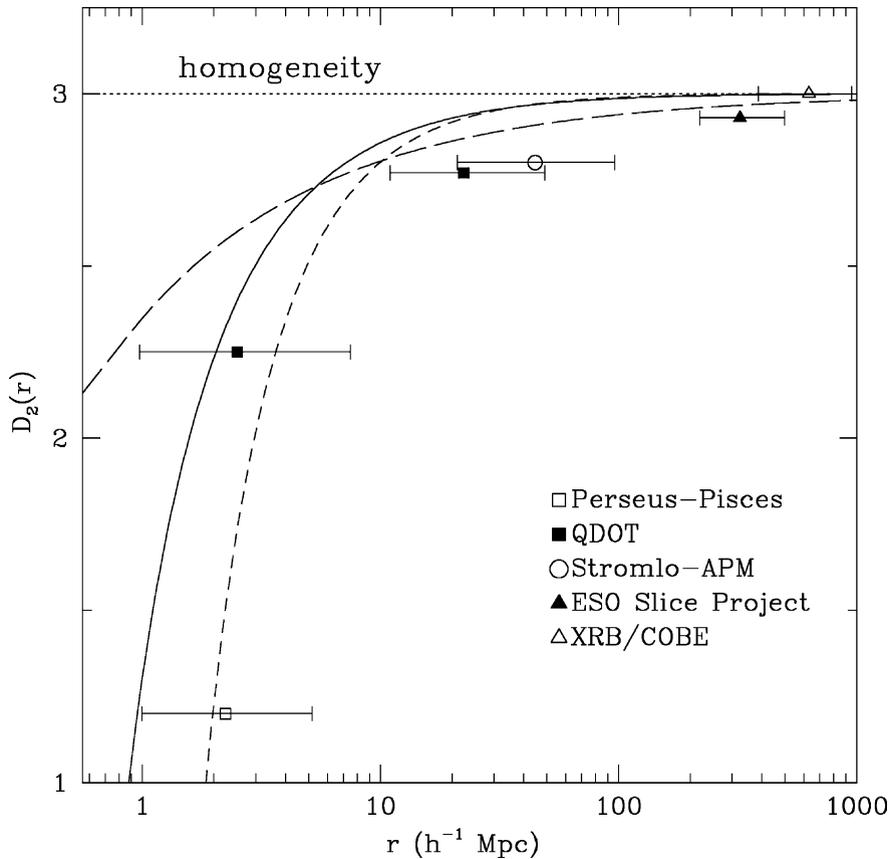


Fig. 2. The correlation dimension vs. scale, for various surveys [2], and for the present model, for $a = 0.65$ and $\beta = 1.0$ (solid line); $a = 1.60$ and $\beta = 0.8$ (dashed line); $a = 0.28$ and $\beta = 2.0$ (long dashed line) with $L = 100 h^{-1} \text{Mpc}$ for all cases.

the entropic parameter and the dynamics of mass clustering in the Universe.

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