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SMALL DEVIATIONS FROM GAUSSIANITY AND THE GALAXY CLUSTER ABUNDANCE EVOLUTION

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ABSTRACT

In this paper, we raise the hypothesis that the density fluctuation field, which originates the growth of large-scale structures, is a combination of two or more distributions, instead of assuming that the observed distribution of matter stems from a single Gaussian field produced in the very early universe, as is widely accepted. By applying the statistical analysis of finite-mixture distributions to a specific combination of Gaussian plus non-Gaussian random fields, we studied the case in which just a small departure from Gaussianity is allowed. Our results suggest that even a very small level of non-Gaussianity may introduce significant changes in the cluster abundance evolution rate.

Subject headings: cosmology: theory — galaxies: clusters: general — large-scale structure of universe

1. INTRODUCTION

In general, the problem of structure formation is associated with the gravitational growth of small density fluctuations generated by physical processes in the very early universe. Also, these fluctuations are supposed to build a Gaussian random field (GRF), where the Fourier components δ_k have independent, random, and uniformly distributed phases. Such a condition means that phases are noncorrelated in space and ensures that the statistical properties of the GRF are completely specified by the two-point correlation function or, equivalently, by the power spectrum $P(k) = |\delta_k|^2$, which contains information on the density fluctuation amplitude of each scale k. This makes the choice of a GRF the simplest initial condition for structure formation studies from the mathematical point of view. At the same time, the GRF simplicity is vindicated by a great number of inflationary models that predict a nearly scale-invariant spectrum of Gaussian density perturbations from quantum-mechanical fluctuations in the field that drives inflation (Guth & Pi 1982). Likewise, the central limit theorem guarantees a GRF if a wide range of random physical processes acts on the distribution of matter in the early universe.

However, a number of mechanisms can generate non-Gaussian density fluctuations. For instance, they arise (1) in some inflation models with multiple scalar fields (e.g., Salopek, Bond, & Bardeen 1989); (2) after phase transitions when different types of topological defects can be formed (Kibble 1976); (3) by any discrete, random, distributed seed masses like primordial black holes and soliton objects (Sherrer & Bertschinger 1991); and (4) in astrophysical processes during the nonlinear regime where early generations of massive stars produce shocks which sweep material onto giant blast waves triggering formation of large-scale structure (Ostriker & Cowie 1981). Thus, in order to better understand the process of structure formation, it is necessary to investigate the possibility of the non-Gaussian statistics contribution to the density fluctuation field.

Because of the difficulty of working with generic statistical models, the usual approach is to examine specific classes of non-Gaussian distributions. Examples of these efforts are the studies carried out by (1) Weinberg & Cole (1992), who studied non-Gaussian initial conditions generated by a range of specific local transformations of an underlying Gaussian field; (2) Moscardini et al. (1991), who investigated whether non-Gaussian initial conditions can help to reconcile the cold dark matter (CDM) models with observations; and (3) Koyama, Soda, & Taruya (1999), who used data on the abundance of clusters at three different redshifts to establish constraints on structure-formation models based on χ^2, non-Gaussian fluctuations generated during inflation.

In this work, we propose a new approach to this problem, exploring the hypothesis that initial conditions for structure formation do not build a single GRF but a combination of different fields, produced by different physical mechanisms, whose resultant effect presents an arbitrarily small departure from the strict Gaussianity. The paper is organized as follows: in § 2, we introduce the statistical analysis of finite mixture distributions and present a two-component mixture model; in § 3, we apply the model to the cluster abundance evolution; in § 4, we summarize and discuss our results.

2. MIXTURE-DISTRIBUTION MODELS: THE POSITIVE-SKEWNESS CASE

Suppose the density fluctuation field, given by the density contrast $\delta = [\rho(r) - \bar{\rho}]/\bar{\rho}$, is a random variable which takes values in a sample space $\mathfrak{R}$ and that its distribution can be represented by a probability-density function of the form

$$p(\delta) = \alpha_1 f_1(\delta) + \dots + \alpha_k f_k(\delta) \quad (\delta \in \mathfrak{R}) , \qquad (2.1)$$

where $\alpha_j > 0, j = 1, \dots, k, \alpha_1 + \dots + \alpha_k = 1$, and

$$f_j(\delta) \geq 0 \; ; \quad \int_{\mathfrak{R}} f_j(\delta) d\delta = 1 \; ; \quad j = 1, \dots, k \; .$$

When this happens, we say that δ has a finite-mixture distribution defined by equation (2.1), where the components of the mixture are $f_1(\delta), \dots, f_k(\delta)$, and the mixing weights are $\alpha_1, \dots, \alpha_k$ (e.g., Titterington, Smith, & Makov 1985). Note that we are not using here the central-limit theorem. Mathematically, this will be valid only when $k \to \infty$ and the weights have similar values, so that one process has no more importance than the others. We are not making these hypotheses here, and, consequently, the summation of processes will not necessarily converge to a Gaussian.

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Statistical evidence for a small level of non-Gaussianity in the anisotropy of the cosmic background radiation temperature has been found in the *COBE* 4-year maps (e.g., Ferreira, Magueijo, & Gósrki 1998; Pando, Valls-Gabaud, & Fang 1998; Magueijo 2000). Non-Gaussian statistics are also expected in the framework of biased models of galaxy formation (Bardeen et al. 1986). In this case, analytical arguments show that non-Gaussian behavior corresponds to a threshold effect superposed on the Gaussian background (Politzer & Wise 1984; Jensen & Szalay 1986). In the same way, hybrid models show that it is possible for structure to be seeded by a weighted combination of adiabatic perturbations produced during inflation and active isocurvature perturbations produced by topological defects generated at the end of the inflationary epoch (e.g., Battye & Weller 1998). Thus, a very compelling way to simplify our model is to apply equation (2.1) to the combination of only two fields: a GRF plus a second field, where the latter will represent a small departure from the strict Gaussianity. This can be posed as

$$p(\delta) = \alpha f_1(\delta) + (1-\alpha) f_2(\delta) \ . \tag{2.2}$$

The first field will always be the Gaussian component, and a possible effect of the second component is to modify the GRF to have positive and/or negative tails. The parameter α in equation (2.2) allows us to modulate the relative importance between the two components. It represents an arbitrarily small departure from the strict Gaussianity and can be the result of some primordial mechanism acting on the energy distribution. Such a two-component random field can be generated by taking $\delta_k^2 = P(k)\nu^2$, where ν is a random number with distribution given by equation (2.2). Then we have

$$\langle \delta^2(r) \rangle = \frac{V}{(2\pi)^3} \int_k P(k) \times \left[\int_\nu [\alpha f_1(\nu) + (1-\alpha) f_2(\nu)] \nu^2 d\nu \right] d^3k \ , \tag{2.3}$$

where V is the volume of an arbitrarily large region of the universe. The quantity in the brackets will be defined as the mixture term

$$T_{\rm mix} \equiv \int_\nu [\alpha f_1(\nu) + (1-\alpha) f_2(\nu)] \nu^2 d\nu \ , \tag{2.4}$$

so that $P(k)_{\rm mix} \equiv P(k) T_{\rm mix}$ for the case in which α is not scale dependent. In the same way, the rms mass overdensity within a certain scale R will be $\sigma^2(R)_{\rm mix} \equiv \sigma^2(R) T_{\rm mix}$.

As an illustration, in this work we explore the case of a positive-skewness model, where the second field adds to the Gaussian component a positive tail representing a number of rare peaks in the density fluctuation field. A simple way to obtain this effect is to take the well-known lognormal distribution as the second component. Besides its mathematical simplicity, this distribution seems to play an important role over the nonlinear regime for a wide range of physical scales (e.g., Coles & Jones 1991; Plionis & Vardarini 1995; Bi & Davidsen 1997). Accordingly, our mixture becomes

$$f_1(\nu) = \frac{1}{\sqrt{2\pi}} e^{-\nu^2/2} \ , \quad f_2(\nu) = \frac{1}{\nu\sqrt{2\pi}} e^{-(\ln \nu)^2/2} \tag{2.5}$$

(for the case of mean zero). Introducing equation (2.5) into equation (2.4), we find

$$T_{\rm mix} = \int_\nu \left[\frac{\alpha}{\sqrt{2\pi}} e^{-\nu^2/2} + \frac{(1-\alpha)}{\nu\sqrt{2\pi}} e^{-(\ln \nu)^2/2} \right] \nu^2 d\nu \ . \tag{2.6}$$

Resolving this integral, we have

$$T_{\rm mix} = \left[\alpha + \frac{e^2}{2} (1-\alpha) \right] . \tag{2.7}$$

Hence, if $\alpha \approx 1$, then $P(k)_{\rm mix} \approx P(k)$ and $\sigma^2(R)_{\rm mix} \approx \sigma^2(R)$, which means that a sufficiently small contribution of the second field leaves the amplitude and shape of the power spectrum and the mass fluctuation practically unchanged.

3. CLUSTER ABUNDANCE EVOLUTION

The correct framework to describe the evolution of nonlinear objects in the context of this model requires a generalization of the Press & Schechter formalism (Press & Schechter 1974) in order to take into account the second field. Assuming that only regions with $\nu > \nu_c$ will form gravitationally bound objects with mass larger than M by the time t, the fraction of these objects can be calculated through

$$F(M) = \int_{\nu_c}^{\infty} p(\nu) d\nu \ , \tag{3.1}$$

where $\nu = \delta/\sigma_R$. This quantity is transformed into the comoving number density of objects with mass between M and $M + dM$ by taking $\partial F/\partial M$ and dividing it by (M/ρ_b). Thus,

$$n(M)dM = 2 \frac{\rho_b}{M} \frac{\partial}{\partial M} \left[\int_{\nu_c}^{\infty} p(\nu) d\nu \right] dM \ , \tag{3.2}$$

where ρ_b is the background density, and the number 2 comes from the correction factor $[\int_0^\infty p(\nu)d\nu]^{-1} = 2$, which takes into account all the mass of the universe. If $p(\nu)$ is given by equation (2.2), then equation (3.2) can be written as

$$n(M)dM = 2 \frac{\rho_b}{M} \frac{\partial}{\partial M} \left[\int_{\nu_c}^{\infty} [\alpha f_1(\nu) + (1-\alpha) f_2(\nu)] d\nu \right] dM \ . \tag{3.3}$$

Now, introducing equation (2.5) into equation (3.3) we have

$$n(M)dM = \sqrt{\frac{2}{\pi}} \left(\frac{\rho_b}{M} \right) \left[\alpha \left(\frac{\partial \nu_c}{\partial M} \right) e^{-\nu_c^2/2} + (1-\alpha) \left(\frac{\partial \ln \nu_c}{\partial M} \right) e^{-(\ln \nu_c)^2/2} \right] dM \ . \tag{3.4}$$

Following Sasaki (1994), we rewrite equation (3.4) to give the density of objects with mass in the range dM about M which virialize at the redshift z and survive until the present epoch without merging with other systems. It becomes

$$n(M, z) = F(\Omega) \left(\frac{M}{M_*(z)} \right)^{(n+3)/3} \sqrt{\frac{2}{\pi}} \left(\frac{\rho_b}{M^2} \right) \times \frac{(n+3)}{6} [\alpha A(M, z) + (1-\alpha) B(M, z)] \ , \tag{3.5}$$

where

$$F(\Omega) = \frac{5}{2}\,\Omega\left[\frac{(1+\frac{3}{2}\Omega)}{(1+\frac{3}{2}\Omega+\frac{5}{2}\Omega z)^2}\right],$$

$$A(M, z) = \left(\frac{M}{M_*(z)}\right)^{(n+3)/6} \exp\left[-\frac{1}{2}\left(\frac{M}{M_*(z)}\right)^{(n+3)/3}\right],$$

$$B(M, z) = \exp\left\{-\frac{1}{2}\ln\left[\frac{M}{M_*(z)}\right]^{(n+3)/6}\right\}^2,$$

$$M_*(z) = M_*(1+z)^{-6/(n+3)} .$$

Equation (3.5) allows us to compare the cluster abundance evolution with observational data. Clusters, as the most massive collapsed structures, correspond to rare peaks in the primordial density field, and so their abundance is sensitive to the occurrence of non-Gaussianity in the density fluctuation distribution. Also, cluster evolution provides a constraint on the amplitude of the mass fluctuation at 8 h^{-1} Mpc scale, σ_8, and on the cosmological density parameter, Ω_m, through the relation $\sigma_8\,\Omega_m^{0.5} \simeq 0.5$ (e.g., Henry & Arnaud 1991; Pen 1998). In a recent work, Bahcall (1999) shows that several independent methods based on cluster data indicate a low-mass density in the universe, $\Omega_m \simeq 0.2$, and in consequence, $\sigma_8 \simeq 1.2$, breaking the degeneracy between these parameters.

Here we compare the behavior of the cluster abundance evolution given by equation (3.5) with data compiled by Bahcall & Fan (1998). As an example, we plot in Figure 1*a* some fits to the observational data for two different values of Ω_m (0.2 and 1.0). Note that our model is very sensitive to the parameter α. Even for $(1-\alpha) \sim 10^{-3}$–10^{-4} (i.e., almost Gaussian initial conditions), the curves diverge significantly from the strict Gaussian cases. This means that even very small deviations from Gaussianity may introduce a significant change in the cluster abundance. Actually, the presence of the second field tends to slow down the cluster abundance evolution at high redshifts. In the case of $\Omega_m = 1.0$ this effect is dramatic for $z > 0.3$, while in the case of $\Omega_m = 0.2$ the difference is less pronounced, and it is clearer for $z \gtrsim 0.6$. Indeed, by plotting the 68% confidence limits around the curve $\Omega_m = 0.2$, we see that Gaussian and non-Gaussian models are not clearly distinguishable for $z \leq 1$ (see Fig. 1*b*). This is related to the small number of observational points and, possibly, to the simplicity of our model. However, even considering these caveats, our results seem to indicate that small deviations from the strict Gaussianity may play an important role in the cluster abundance evolution.

4. SUMMARY AND DISCUSSION

We presented the first results of a study concerned with small deviations from Gaussianity in the primordial density field. Using very simple arguments, we developed a model based on the combination of two random fields in order to take into account the non-Gaussianity effects. This model may be physically motivated in the context of hybrid models, as well as in the framework of biased scenarios for structure formation. The weighted combined field involves a parameter α which modulates the relative importance of its components. For $\alpha \approx 1$, we preserve the amplitude and shape of $P(k)$ and $\sigma(R)$ almost the same as in the Gaussian case. At the same time, our results suggest that even very small values of $(1-\alpha)$ can introduce a significant change in the cluster abundance evolution. This effect seems to be stronger in high-density universes (at $z \leq 1$) than in low-density universes, where the effect probably becomes more important at higher redshifts.

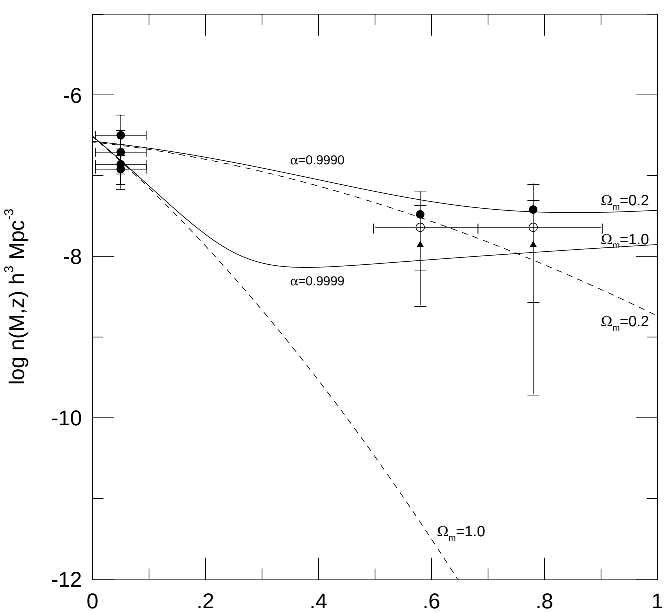

FIG. 1*a*

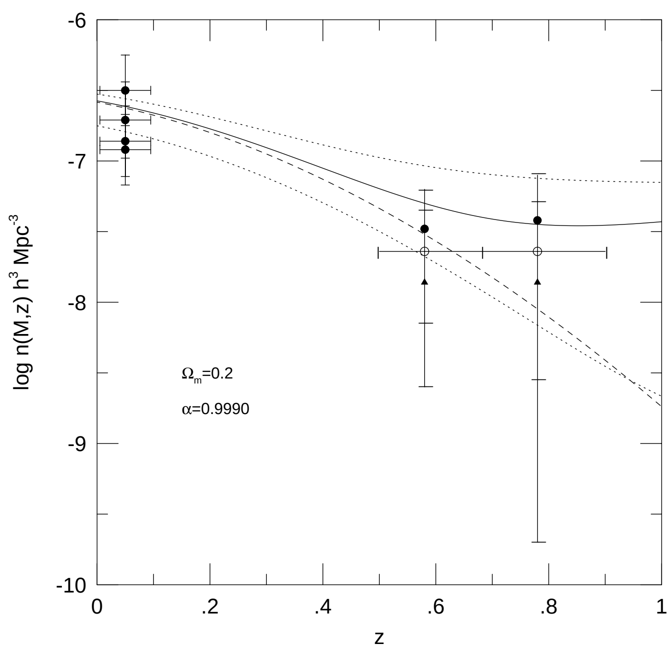

FIG. 1*b*

FIG. 1.—(*a*) Galaxy cluster abundance in the two-component model for $\Omega_m = 0.2$ with $\alpha = 0.9990$ (*solid*) and $\alpha = 1$ (*dashed*) and for $\Omega_m = 1.0$ with $\alpha = 0.9999$ (*solid*) and $\alpha = 1$ (*dashed*). The curves, normalized at $z = 0$, correspond to $n = -1.0$, $M_* = 10^{14}\,h^{-1}\,M_\odot$, and $M > 8 \times 10^{14}\,h^{-1}\,M_\odot$. The observational points were taken from Bahcall & Fan (1998). (*b*) Galaxy cluster abundance in the two-component model for $\Omega_m = 0.2$ with $\alpha = 0.9990$ (*solid*) and $\alpha = 1$ (*dashed*). The dotted lines are the 68% confidence limits around the non-Gaussian fit. The observational points were taken from Bahcall & Fan (1998).

The model has some drawbacks. First, it is dependent on the choice and amplitude of the second component of the combined field. Our choice of the lognormal function had a mathematical criterion of simplicity. A detailed investigation of the use of different distribution functions as the second component will be the subject of future works. However, the reasonable agreement between the model and the data gives some support to our arbitrary choice. Other possible limitation of this work comes from the use of the analytical approximation to the density of nonlinear objects following Sasaki (1994). A more accurate description of the cluster abundance evolution requires the utilization of numerical methods. But Blain & Longair (1993), also working in the Press & Schecter (1974) framework, found results numerically similar to Sasaki's, so we think the use of this analytical approximation does not introduce any systematic error. Finally, we should keep in mind that our results are preliminary and that both theoretical and observational efforts are necessary in order to confirm or disprove the hypothesis that the primordial density field can be described as a slightly non-Gaussian distribution.

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