Fundamentos de Cosmologia AST-413 - 4

Lecture 8 The Cosmic Microwave Background

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The Planck one-year all-sky survey

eesa

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Outline

- Lecture 1: Introduction to the Cosmic Microwawe Background
- Lecture 2: The CMB spectrum
- Lecture 3: The CMB angular distribution and polarization
- Lecture 4: Secondary anisotropies: the Sunyaev-Zeldovich effect, CMB lensing and foreground emission
- Lecture 5: CMB observations and instrumentation













The Cosmic Microwave Background WMAP satellite (2003)



Modern cosmology is presently based upon: General relativity, the cosmological "principle" and particle physics

- Observations that support the theoretical framework above:
 - expansion of the Universe
 - The synthesis of light elements
 - cosmic microwave background (CMB)



The Cosmic Microwave Background

- Oldest electromagnetic signal we can observe!
- Picture of the Universe at 380.000 years after the Big Bang
- Described by a blackbody spectrum at T = 2,725 (± 0,001) K
- CMB temperature fluctuations, $\Delta T \sim 10^{-5} \kappa$, linked to primordial density fluctuations, shed light on the physics of large structure formation in the Universe
- CMB observables: spectral distribution, angular distribution (anisotropies) and polarization



A few interesting numbers...

- T = 2,725 ± 0,001 Kelvin (Mather et al. 1999, ApJ, 512, 511).
- Integral characteristics of the radiation
 - Energy density: $u_{\gamma}=\sigma T^4=4.244 \times 10^{-13} \text{ erg/cm}^3$
 - Number density: n = 0.244 $(kT_0/\hbar c)^3 = 414 \text{ cm}^{-3}$
 - Density: $\rho = 4,6417 \times 10^{-34} (T/2.725) g.cm^{-3}$ 1 out of 3 photons from open TV is a CMB photon!!!
 - Entropy: $4/3 u_{\gamma}/kT_0 = 1.496 \times 10^3 \text{ cm}^{-3}$

 - $_{\circ}~P_{CMB} \sim 10^{-18}~W$
 - Speed of the Sun respect to the CMB = $369,3 \pm 2,5 \text{ km/s}$



1964 Cosmic Microwave Background

1992 Anisotropies



Physics Nobel Prize (1978)

Physics Nobel Prize (2006)



Radioastronomy

1933







- 1934 R. Tolman shows that CMB in an expanding Universe cools down but keep its thermal distribution and remains as a blackbody
- 1941 A McKellar was attempting to measure the average temperature of the interstellar medium, and reported the observation of an average bolometric temperature of 2.3 K based on the study of interstellar absorption lines.
- 1946 R. Dicke predicts ".. radiation from cosmic matter" at <20 K but did not refer to background radiation[14]
- 1948 G. Gamov calculates a temperature of 50 K (assuming a 3-billion-year old Universe), commenting it ".. is in reasonable agreement with the actual temperature of interstellar space", but does not mention background radiation.
- 1948 R. Alpher and R. Herman estimate "the temperature in the Universe" at 5 K. Although they do not specifically mention microwave background radiation, it may be inferred.
- 1950 R. Alpher and R. Herman re-estimate the temperature at 28 K.
 1950 George Gamow estimates 7 K.

- 1955 Émile Le Roux of the Nançay Radio Observatory, in a sky survey at λ =33 cm, reported a near-isotropic background radiation of 3±2 K.
- 1956 George Gamow estimates 6 K.
- $_{\circ}$ 1957 T. Shmaonov reports that "the absolute effective temperature of the radioemission background ... is 4±3K". It is noted that the "measurements showed that radiation intensity was independent of either time or direction of observation... it is now clear that Shmaonov did observe the cosmic microwave background at a wavelength of 3.2 cm".
- 1960 Robert Dicke re-estimates a MBR (microwave background radiation) temperature of 40 K
- 1964 A. G. Doroshkevich and Igor Novikov publish a brief paper, where they
 name the CMB radiation phenomenon as detectable.
- 1964-65 A. Penzias and R. Wilson measure the temperature to be approximately 3 K. R. Dicke, P. J. E. Peebles, P. G. Roll, and D. T. Wilkinson interpret this radiation as a signature of the big bang.

- 1966 R. Sachs and A. Wolfe predict amplitude fluctuations in the CMB created by variations of the gravitational potential between the last scattering surface and the observer
- 1967 M. Rees publishes the first calculations about CMB polarization
- 1969 R.A. Sunyaev e Y. B. Zel'dovich study the inverse Compton scattering of CMB photons caused by hot electrons, defining the basis of the so-called Sunyaev-Zeldovich effect
- 1978 A. Penzias and R. Wilson receive the Nobel Prize for discovering the CMB
- 1983 RELIKT-1 Soviet CMB anisotropy experiment was launched.
- Igo FIRAS/COLE measures the black body form of the CMB spectrum with exquisite precision.
- Igentists who analyzed data from DMR announce the discovery of the primary temperature anisotropy.
- Igginaria 1999 First measurements of acoustic oscillations in the CMB anisotropy angular power spectrum from the TOCO, BOOMERANG, and Maxima Experiments.

- 2000 Results from the BOOMERANG team state that the Universe is, essentially, Euclidian.
- 2001 The WMAP (Wilikinson Microwave Anisotropy Probe) was launched by NASA, Its measurements start to fullfil the hope of measuring/determining cosmological parameters with precision < 10% ⇒ Precision cosmology
 </p>
- 2002 Polarization discovered/measured by DASI.
- 2004 E-mode polarization spectrum obtained by the CBI.
- 2005 <u>Ralph A. Alpher</u> is awarded the <u>National Medal of Science</u> for his groundbreaking work in nucleosynthesis and predictions supporting the Hot Big Bang model.
- 2006 Two of COBE's principal investigators, <u>George Smoot</u> and <u>John Mather</u>, received the <u>Nobel Prize in Physics</u> in 2006 for their work on precision measurement of the CMBR.
- 2009 Planck (Max Planck Surveyor, previously COBRAS/SAMBA), the 3rd generation satellite dedicated to CMB studies, was launched by ESA on May, 14 and has released its first results in January 2011. The CMB data from Planck is expected by April 2013

 The Golden Decade - 2010 - 2020 Results from the BOOMERANG team state that the Universe is, essentially, Euclidian.



The standard scenario

- Friedmann equations and Robertson-Walker metric describe the space-time where CMB evolves
- Boltzmann equations describe the interplay between all the components and how they affect the growth of perturbations
- Theory of cosmological perturbations describes the evolution of density perturbations from the linear into non-linear regime, setting ground for structure formation scenarios.



Primordial Perturbations: fluid at redshift z < 10⁹

- Matter dominated epoch
 - Photons
 - Nearly massless neutrinos: free-streaming (no scattering) after neutrino decoupling at z $\sim 10^9$
 - Baryons + electrons: tightly coupled to photons by Thomson scattering
 - Dark Matter: assume cold. Coupled only via gravity.
 - Dark energy: probably negligible before t < 6×10⁹ years



General regular linear primordial perturbation

General regular perturbation



+ irregular modes, neutrino n-pole modes, n-Tensor modes Rebhan and Schwarz: gr-qc/9403032 + other possible components, e.g. defects, magnetic fields, exotic stuff...

The early Universe is, for most purposes, in thermal equilibrium, coming from:

Tinter << Texp

 $n_X << n_\gamma$

- Departure from equilibrium (violates above condition): not common, but with interesting consequences
 - formation of light elements (T \sim 0.1 MeV)
 - recombination of e- and p to form neutral H (T~0.2 eV)
 - formation of dark matter



Previous to CMB creation...

- Nuclesynthesis during radiation dominated Universe
- $_{\odot}$ Matter domination at \sim 10^4 years
- $_{\odot}$ Cooling and recombination at \sim 10 5 years
- $_{\odot}$ Decoupling of matter and radiation (CMB creation) $~\sim$ 3.8 \times 10^5 years

CMB fluctuations

Large scale: deviations from the FL geometry

Small scale: fluctuations in the energy density of the baryon/radiation plasma

The tool: Boltzmann equation! In the absence of interactions, annihilation of species 1 due to reaction with species 2, yields species 3 and 4 (reversible reaction): 1 + 2 4 3 + 4

$$\begin{aligned} \frac{1}{a^3} \frac{d(n_1 a^3)}{dt} &= \Pi_{i=1}^4 \int \frac{d^3 p_i}{(2\pi)^3 2E_i} \times \\ &\quad (2\pi)^4 \ \delta^3(p_1 + p_2 - p_3 - p_4) \times \\ &\quad \delta(E_1 + E_2 - E_3 - E_4) |M|^2 \times \\ &\quad \delta(E_1 + E_2 - E_3 - E_4) |M|^2 \times \\ &\quad \left\{ f_3 f_4 [1 \pm f_1] [1 \pm f_2] - f_1 f_2 [1 \pm f_3] [1 \pm f_4] \right\} \\ f_{B(+)F(-)} &= \frac{1}{e^{E/T} \pm 1} \end{aligned}$$



The tool: Boltzmann equation!

Approximations:

- + Kinetic equilibrium (enforced by scattering processes)
- + Cases of interest: T << E μ , minimizes the effects of Fermi blocking/Bose enhancement. when set=0,

defines the equilibrium condition

$$\frac{1}{a^3} \frac{d(n_1 a^3)}{dt} = n_1^0 n_2^0 < \sigma v > \left\{ \frac{n_3 n_4}{n_3^0 n_4^0} - \frac{n_1 n_2}{n_1^0 n_2^0} \right\} \\ < \sigma v > = \Pi_{i=1}^4 \int \frac{d^3 p_i}{(2\pi)^3 2E_i} e^{-(E_1 + E_2)/T} \times \\ (2\pi)^4 \ \delta^3(p_1 + p_2 - p_3 - p_4) \times \\ \delta(E_1 + E_2 - E_3 - E_4) |M|^2$$

 Boltzmann-Einstein equations govern the evolution of matter inhomogeneities and temperature anisotropies



- Solution of Boltzmann equations for dark matter, baryons, photons and neutrinos demand the computation of colision terms. They apply to photons through Compton scattering by electrons and to electrons and protons through Coulomb scattering
- Necessary terms are the gravitational Newtonian potential (Ψ) , the perturbation to the spatial curvature (Φ) and the perturbations to the photons caused by the metric (Θ)



Simplified version of Boltzmann eq.; RHS represents the colision terms

$$f(\vec{x}, p, \hat{p}, t) = \left[\exp\left\{ \frac{p}{T(t)[1 + \Theta(\vec{x}, \hat{p}, t)]} \right\} \right]^{-1}$$

- Θ is the perturbation in temperature distribution T, and is what we usually understand as $\Delta T/T$!!!
- In a uniform Universe, T is homogeneous and decreases only with time t, but variations in the direction of propagation of photons (p) and inhomogeneities in space (x) produce second order effects (Θ)
- Similar procedure for the other components: dark matter, baryons and neutrinos

Boltzmann eqs. for all Species P subscript - polarization Photons . – conformal time derivative $\dot{\Theta} + i\kappa\mu\Theta = -\dot{\Phi} - i\kappa\mu\Psi - \dot{\tau}\left|\Theta_0 - \Theta + \mu v_b - \frac{1}{2}P_2(\mu)\Pi\right|$ $\Pi = \Theta_2 + \Theta_{P_2} + \Theta_{P_0}$ $\dot{\Theta}_P + i\kappa\mu\Theta_P = -\dot{\tau}\Big[-\Theta_P + \frac{1}{2}(1-P_2(\mu))\Pi\Big]$ Dark matter $\dot{\delta} + i\kappa v = -3\dot{\Phi}, \quad \dot{v} + \frac{a}{c}v = -i\kappa\Psi$ Baryons $\dot{\delta}_b + i\kappa v_b = -3\dot{\Phi}, \quad \dot{v}_b + \frac{\dot{a}}{a}v_b = -i\kappa\Psi + \frac{\dot{\tau}}{R}\left[v_b + 3i\Theta_1\right]$ Neutrinos $\dot{N} + i\kappa\mu N = -\dot{\Phi} - i\kappa\mu\Psi$

$$\Theta_l \equiv \frac{1}{(-i)^l} \int_{-1}^1 \frac{d\mu}{2} P_l(\mu) \Theta(\mu)$$

 $\dot{\tau} = n_e \sigma_T a$ $\mu = \hat{p} \bullet \hat{k}$



Further info

- Modern Cosmology. S. Dodelson (2005)
- The Cosmic Microwave Background. R. Durrer (2008)
- For Boltzmann eqs.:
 - Papers from Efsthatiou (1990), Ma & Bertschinger (1995), Kosowsky (1996, for polarization)
- For gauge issues: Kodama & Sasaki (1984), Mukhanov,
 Feldman & Brandenberger (19932)

THE CMB BLACKBODY SPECTRUM



The blackbody

spectrum:

energetics!!!

The CMB spectrum



Distortions caused by Phase transition (Theories of Unification) Relic particle decay (Dark Matter) Reionization (first stars)







CMB Black body and cosmology

A blackbody with T=2.725K consists of ~ 414 γ /cm³ This is $\sim 10^9$ higher than the average density of matter in the universe, so CMB photons are the most abundant particles in the universe The Black Body nature of the CMB is a direct confirmation that the universe underwent a hot early phase (no mention about the beginning!!!!) The primordial hot phase is the way to produce such a perfect blackbody spectrum, filling the sky with a rdiation field which is isotropic in 1 part in 10⁵



What can we learn from the CMB spectrum?

- Spectrum measurements can tell us about
 - Reionization
 - Departures from thermodynamical equilibrium
- Some assumptions
 - Matter-radiation coupling: $e^+ + e^- \Leftrightarrow 2 \gamma$
 - Coulomb scattering
 - $_{\odot}$ Interaction of the radiation field with e^ and baryons



What can we learn from the CMB spectrum?

- Photon creation or energy change can be accomplished through
 - Thermal Bremsstrahlung (e⁻, p)
 - Compton scattering
 - Double Compton scattering, where a 2^{nd} photon is produced in (e-, γ) collision



The Planckian function

$$B_
u(T) = rac{2h
u^3}{c^2}rac{1}{e^{h
u/\kappa T}-1} \ B_
u(T) = 2h
urac{
u^2}{c^2}rac{1}{e^{h
u/\kappa T}-1} \ Polarization states Occupation number$$

Rayleigh-Jeans limit (x << 1)

$$B_{\nu}(T) \sim 2kT \frac{\nu^2}{c^2}$$

 $B_{\nu}(T)$

Wien limit (x >> 1)

$$\sim \frac{2h\nu^3}{c^2}e^{-x}$$
 $x =$





$$u_{\lambda}d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda\kappa T} - 1}$$

Due to expansion and the redshift:

$$\frac{\lambda}{\lambda_0} = \frac{a}{a_0}; \quad \frac{u_\lambda d\lambda}{u_{\lambda_0} d\lambda_0} = \left[\frac{a}{a_0}\right]^2$$



How does a black body evolve in an expanding Universe?

$$u_{\lambda_{0}}d\lambda_{0} = \left[\frac{a}{a_{0}}\right]^{4}u_{\lambda}d\lambda = \left[\frac{a}{a_{0}}\right]^{4}\frac{8\pi hc}{\lambda_{0}^{5}\left[\frac{a}{a_{0}}\right]^{5}\left(e^{hc/\lambda_{0}\left[a/a_{0}\right]\kappa T}-1\right)}d\lambda_{0}\left[\frac{a}{a_{0}}\right]$$

$$\Rightarrow u_{\lambda}d\lambda = \frac{8\pi hc}{\lambda_{0}^{5}\left(e^{hc/\lambda_{0}\left[a/a_{0}\right]\kappa T}-1\right)}d\lambda_{0}$$

$$T = T_{0}\frac{a_{0}}{a}$$

$$T = T_{0}\frac{a_{0}}{a}$$

- So, the Universe evolves and cools down, and the black body still remains the same
- Photons from primordial hot phases should still be around, but with lower energy (as a low-T black body)




Noterdaeme et al. (2011)



What else can we learn from the CMB spectrum?

- Neutral atoms seldom interact with microwave photons
 - Ions and e- in the primordial plasma interact via Thomson scattering \rightarrow changes direction but not photon frequency
 - Thomson scattering does not produce a thermal spectrum!!



What can we arrive to a black body spectrum?

- Under which conditions a radiation field, initially non-Planckian, relax towards a Planckian shape? Two conditions are necessary:
 - A mechanism to create photons and/or redistribute its energy
 - The interaction rate of these mechanisms MUST BE LARGER than the expansion rate of the Universe.
- In the primordial Universe, matter and radiation in thermal equilibrium create a Planckian spectrum AND expansion DOES NOT CHANGE its Planckian character











Distortions in the black body spectrum

- The blackbody spectrum of CMB was created in the blackbody photosphere at redshifts z > 2 x 10^6 .
- Universe dense and hot enough \Rightarrow complete thermal equilibrium; any perturbation away from the blackbody spectrum was suppressed exponentially.
- New physics (e.g., annihilation and decay of dark matter) can add energy and photons to CMB at redshifts $z \sim 10^5$ and result in a Bose-Einstein spectrum with a non-zero chemical potential (μ) \Rightarrow **DISTORTION**
- Processes: double Compton process, comptonization and bremsstrahlung
- Triple degeneracy in standard cosmology, i.e., the μ and y distortions from adiabatic cooling of baryons and electrons, Silk damping and annihilation of thermally produced WIMP dark matter are of similar order of magnitude (~ $10^{-8} 10^{-10}$).



Blackbody distortions





What causes the distorti

A state of the chemical pot of the terms of the component of the chemical pot of the terms of the component of the

 $y = \int \frac{\kappa_B T_e}{m_e c^2} \sigma_T n_e dl = \int d\tau_e \frac{\kappa_B T_e}{m_e c^2}$

RECOMBINATION AND DECOUPLING







Figure 8.4 An observer is surrounded by a spherical last scattering surface. The photons of the CMB travel straight to us from the last scattering surface, being continuously redshifted.





The sequence

- CMB photons are the original photons that were coupled to the matter since after inflation.
- Matter, in this case, start from quarks & gluons, at a very early time, and became later neutrons & protons
- $_{\rm 0}$ Photons and matter interacted closely due to Thomson scattering of e^ and γ
- Recombination (in fact, COMBINATION...) happened when temperature was low enough to allow electrons and protons to COMBINE into a hydrogen atom
- Recombination happens (formally) when $n_{ions} = n_{neutral}$



- Photon decoupling happens after e⁻ are removed from the environment, extinguishing the primordial plasma
- The Universe becomes "transparent"; before it was "opaque"
- Formally it happens when the photon mean free path becomes larger than the radius of the Universe at that time, or when the rate at which photons interact become smaller than the Hubble expansion rate
- The above define "the last scattering surface", the hypersurface where a photon last interacts with an e⁻
- Since recombination does not happen instantaneously, the "surface" is actually a shell... Recombination starts, decoupling takes place and the beginning and end of these events define the shell



- B. Ryden's treatment includes H only, for simplification, but the formalism is exactly if we consider the real world, where He, D and Li are presente.
- X is the fraction of protons versus neutral H and protons; electrons quick in due to the neutrality requirement!



 For H, the relevant energy scale is the ionization energy E=13.6 eV, which affects the reaction rate below

$H + \gamma \rightleftharpoons p + e^-$

X depends on the balance of the above equation



- During early times, when the Universe was much smaller than today, its temperature was much higher. If we consider the time when $a(t) = 10^{-5}$, $T \sim 3 \times 10^{5} K \approx$ $60 \ eV$, $t \sim 10^{2} yr$
- This is more than enough a temperature to keep the whole environment in an ionized state. In this case, typical scattering reactions such as

$$e^- + \gamma \rightleftharpoons \gamma + e^-$$

- These reactions were responsible for sharing the energy among all components at the time
- Interaction rate at the time was $\Gamma = 5 \times 10^{-6}/s$, or about one scattering/week. This was much slower than the expansion rate at the time (10⁻¹⁰/s), making photons and electrons coupled.



The physics of recombination

- Recombination temperature T=60000K... But <E> = 2.7 kT is not a very good approximation, since the blackbody distribution is A DISTRIBUTION, not an average.
- So we can have a small number of photons with E >> 2.7
 KT in this distribution. Ryden quotes 1/500 with E > 10 kT, 1/3x10⁶ with E > 20 kT and 1/3x10¹⁰ with E > 30 kT
- Since there are ~ 10⁹ photons for each baryon or electron, the chance of an atom being quickly ionized at early times is enormous.
- $\,$ The calculation of the reaction rate for $H+\gamma\rightleftarrows p+e^-$ depends upon T and the photon/baryon ratio!!!



- The process of computing the fractional ionization state of the Universe as a function of temperature and the available species at the time uses the Saha equation and some statistical mechanics.
- For photons: $n_{\gamma} = \frac{2.4041}{\pi^2} \left(\frac{kT}{hc}\right)^3 = 0.2436 \left(\frac{kT}{hc}\right)^3$

 For electrons and protons (with appropriate considerations for statistical weights, masses and energies:

 $\frac{n_H}{n_p n_{e^-}} = \frac{g_H}{g_p g_e} \left(\frac{m_H}{m p m_e}\right)^{3/2} \left(\frac{kT}{2\pi h^2}\right)^{-3/2} exp\left(\frac{[m_p + m_e - m_H]c^2}{kT}\right)$ $= \left(\frac{m_e kT}{2\pi h^2}\right)^{-3/2} exp\left(\frac{Q}{kT}\right)$



- Preparing the above results to be used in the Saha equation, and considering the photon/baryon relation to the number of free protons, we obtain:
- $\frac{1-X}{X} = n_p \left(\frac{m_e kT}{2\pi h^2}\right)^{-3/2} \left(\frac{Q}{kT}\right), \quad \eta = \frac{n_p}{Xn_{\gamma}}$ We can put the equation above in the form of a quadratic equation. Solving it for X, we get: $S(T,\eta) = 3.84\eta \left(\frac{kT}{m_e c^2}\right)^{3/2} \left(\frac{Q}{kT}\right)$ • Considering X=1/2, and η^{\sim} 6x10⁻¹⁰, corresponding to;
 - $T_{rec} = 3760 \text{ K}, z_{rec} = 1380, t_{rec} = 250.000 \text{ years}$



 B. Ryden discusses the non-instantaneous process that makes the last scattering surface actually a shell. Nevertheless, since the drop in the number of free protons is very fast, decopling happpens almost immediately after recombination.



Figure 8.5 Fractional ionization *X* as a function of redshift during the epoch of recombination. A baryon-to-photon ratio $\eta = 6.1 \times 10^{-10}$ is assumed. Redshift decreases, and thus time increases, from left to right.

THE CMB TEMPERATURE FLUCTUATIONS



CMB anisotropies

Anisotropy: dynamics

Distribution of density fluctuations Global cosmological parameters Inflation physics



What can we learn from CMB anisotropies?

- Initial conditions: What types of perturbations, power spectra, distribution function (Gaussian?); => learn about inflation or alternatives. (distribution of ΔT; power as function of scale; polarization and correlation)
- What and how much stuff: Matter densities (Ω_b, Ω_{cdm}) ; neutrino mass (details of peak shapes, amount of small scale damping)
- Geometry and topology: global curvature Ω_{K} of universe; topology (angular size of perturbations; repeated patterns in the sky)
- Evolution: Expansion rate as function of time; reionization, Hubble constant H₀; dark energy evolution w = pressure/density (angular size of perturbations; / < 50 large scale power; polarization)
- Astrophysics: S-Z effect (clusters), foregrounds, etc.



Simple classification

• Origin:

- Intrinsec (pre-decoupling)
 - gravitational potential (SW), local Doppler effects, primordial perturbations
- Extrinsec or secondary (pos-decoupling)
 - Gravitational: Ostriker-Vishniac, Rees-Sciama, lensing, ISW
 - Scattering: SZ effect, reionization

Angular contributions

- Large scale ($\theta \ge 1^{\circ}$) Sachs-Wolfe, mostly gravitational
- Intermediate scales $(1' \le \theta \le 1^\circ)$ mainly from photon dynamics
- Small scales ($\theta \leq 1'$) dissipation of primary anisotropies





polarization, secondary scattering and, for pos-recombination, mainly SZ effect and lensing, will be discussed later...



Solutions

$T(\vec{\kappa},\mu,\eta) = T_0(\eta)[1+\Theta(\vec{\kappa},\mu,\eta)]$

temperature perturbation



Spherical harmonics give the temperature coefficients.

How do we compute the CMB power spectrum? ΔT/T computed from the integration (in conformal time) of

- ΔT/T computed from the integration (in conformal time) of Einstein and Boltzmann eqs. along the line-of-sight photon cone
 - Geometric term: radial eigenfunctions, model-independent
 - Source terms: photon, baryon and metric perturbations
- Polarization spectrum: comes from rotation due to Stokes parameters' determination. It is weaker, includes scalar, vector and tensor components



How do we compute the CMB power spectrum?

 $\frac{\Delta T(\theta,\phi)}{T_{CMB}} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta,\phi)$

> spatial distribution of the amplitudes of the fluctuations

encodes the physical processes imprinted in the radiation produced in the LSS

$$Y_{lm}(\theta,\phi) = \sqrt{\frac{2l+1}{4\pi}} \frac{|l-m|!}{|l+m|!} P_l^m(\cos\theta) e^{im\phi}$$

 $a_{lm}^* = (-1)^m a_{l-m}$

since the temperature is a real quantity...



Boltzmann eqs. for all Species P subscript - polarization Photons . – conformal time derivative $\dot{\Theta} + i\kappa\mu\Theta = -\dot{\Phi} - i\kappa\mu\Psi - \dot{\tau}\left|\Theta_0 - \Theta + \mu v_b - \frac{1}{2}P_2(\mu)\Pi\right|$ $\Pi = \Theta_2 + \Theta_{P_2} + \Theta_{P_0}$ $\dot{\Theta}_P + i\kappa\mu\Theta_P = -\dot{\tau}\Big[-\Theta_P + \frac{1}{2}(1-P_2(\mu))\Pi\Big]$ Dark matter $\dot{\delta} + i\kappa v = -3\dot{\Phi}, \quad \dot{v} + \frac{a}{c}v = -i\kappa\Psi$ Baryons $\dot{\delta}_b + i\kappa v_b = -3\dot{\Phi}, \quad \dot{v}_b + \frac{\dot{a}}{a}v_b = -i\kappa\Psi + \frac{\dot{\tau}}{R}\left[v_b + 3i\Theta_1\right]$ Neutrinos $\dot{N} + i\kappa\mu N = -\dot{\Phi} - i\kappa\mu\Psi$

$$\Theta_l \equiv \frac{1}{(-i)^l} \int_{-1}^1 \frac{d\mu}{2} P_l(\mu) \Theta(\mu)$$

 $\dot{\tau} = n_e \sigma_T a$ $\mu = \hat{p} \bullet \hat{k}$



- $_{\circ}$ If we compute the previous set of equations up to the angular scale of interest (I \sim 3000 and up, in 2020)...
- Huge number of coupled equations (may take days to solve by brute force on the fastest computers)
- Strategies were developed to solve them...
 - Initial code (COSMICS): Bertschinger (1995)
 - Breakthrough (CMBFAST): Seljak & Zaldarriaga (1996)
 - The present (CAMB and CMBEASY): Lewis and Challinor (1999), Doran, Robbes and Müller (2003)
 - A coupled pipeline (COSMOMC): Lewis and Bridle (2002)
 - Many other codes in the market in 2020, based in Python, but CAMB remains the latest reference for all of them



Line-of-sight integration

- Instead of expanding the set of equations in multipoles and integrating them, solve equation for $d\Theta/dt$ and do the multipole expansion at the end!
- Order of magnitude gain in time processing
- \circ Solution for Θ

$$\Theta(\kappa,\mu,\eta_0) = \int_0^{\eta_0} \left\{ -\dot{\Phi} - i\kappa\mu\Psi - \dot{\tau} \Big[\Theta_0 + i\mu v_b - \frac{1}{2} P_2(\mu)\Pi \right\} \times e^{i\kappa\mu(\eta-\eta_0)^{-\tau}} d\eta$$



$$egin{array}{rll} \Theta_l(\kappa,\eta_0)&=&\int_0^{\eta_0}S(\kappa,\eta)j_l[\kappa(\eta-\eta_0)]d\eta\ &&S(\kappa,\eta)&=&gig[\Theta_0+\Psi+rac{1}{4}\Piig]+e^{- au}ig[\dot{\Psi}-ec{2}$$

µdisappears, being absorbed by the spherical Bessel function jı

 $\frac{1}{\kappa}\frac{d}{d\eta}(gv_b) + \frac{3}{4\kappa^2}\frac{d^2}{d\eta^2}(g\Pi)$ • C₁ are, essentially, $\Theta_1^2(\mathbf{x})$, with the contribution of spectrum, defined by the initial conditions

Harrison-Zeldovich spectrum, predicted by inflation...

 Generally speaking (not including normalizations for the amplitude of the fluctuations):

$$C_l = \int \frac{d^3\kappa}{2\pi^2} P(\kappa)\Theta_l^2(\kappa)$$

$$\frac{\kappa^3}{2\pi^2} P(\kappa) = \left(\frac{\kappa}{H_0}\right)^{n-1}, \text{ so}$$
$$C_l = \int_0^\infty \left(\frac{\kappa}{H_0}\right)^{n-1} \Theta_l^2(\kappa) \frac{d\kappa}{\kappa}$$

The CMB power spectrum

Primordial perturbations + later physics



Hu & White, Sci. Am., 290 44 (2004)



From Paul Steinhardt (1995)

Speaking of acoustic waves...





 $\frac{d}{d\eta} \begin{bmatrix} e^{-\tau} (\Theta + \Psi) \end{bmatrix} = e^{-\tau} (\dot{\phi} + \dot{\psi}) & \text{scattering by} \\ \text{moving e-} & \text{radiation density} \\ \text{fluctuations} \\ \text{fluctuations} \\ \text{integrated Sachs-Wolfe} & - \dot{\tau} e^{-\tau} \left(\Psi + \vec{e} \bullet \vec{v}_b + \frac{3}{16\pi} \int d\hat{m} \Theta(\epsilon, \hat{m}) [1 + (\vec{e} \bullet \hat{m})^2] \right)$



(l+1) - accounts for the real contribution
of the alm to the power spectrum (lost
 when the average is taken <>)



WMAP 7-year power spectrum (Jarosik et al 2011)










From Wayne Hu @ http://background.uchicago.edu/~whu/araa/node4.html



Consistency tests



From Paolo de Bernardis (lecture @ INPE's IV Advanced School)





The consistency of the maps from three *independent* experiments, working at very different frequencies and with very different mesurement methods, is the best evidence that the faint structure observed •*is not due to instrumental artifacts* •*has exactly the spectrum of CMB anisotropy, so it is not due to foreground emission* •The comparison also shows the *extreme sensitivity of cryogenic bolometers* operated at balloon altitude (the B03 map is the result of 5 days of observation)

From Paolo de Bernardis (lecture @ INPE's IV Advanced School)



Just for your curiosity...

- Best determination of CMB temperature: TCMB = 2.7255 +/- 0.0006 K (Fixsen et al., ApJ 2009)
- Temperature interval of a CMB anisotropy map: +/- 0.0003 K





CMB polarization



Reionization

Matter distribution in the LSS

•

V band

Primordial gravitational waves

Polarization: second order perturbations

Ka band



SCATTERING!!!!!





Why CMB polarization?

- An inflation phase at E=10¹⁶-10¹⁵ GeV (t=10⁻³⁶-10⁻³³ s) is currently the most popular scenario to explain
 - The origin of our universe
 - The geometry of our universe
 - The origin and morphology of structures in our universe
 - The lack of defects, and the smoothness of the CMB at super-horizon scales.
- Inflation is a predictive theory:
 - 1. Any initial curvature is flattened by the huge expansion: we expect an Euclidean universe.
 - 2. Adiabatic, gaussian density perturbations are produced from quantum fluctuations. This is the physical origin for structures in the Universe.
 - The power spectrum of scalar perturbations is approximately scale invariant, P(k)=Akn⁻¹ with n slightly less than 1.
 - Tensor perturbations produce a background of primordial gravitational waves (PGW)
- 4. can be tested measuring CMB polarization
- 1.,2.,3. have been confirmed already by measurements of CMB anisotropy



Why CMB polarization?

- Linear Polarization of CMB photons is induced via Thomson scattering by quadrupole anisotropy at recombination (z=1100, $t = 1.2 \times 10^{13}$ s).
- In turn, quadrupole anisotropy is induced by
 - Density perturbations (scalar relics of inflation) producing a curl-free polarization vectors field (Emodes)
 - Gravitational waves (tensor relics of inflation) producing both curl-free and curl polarization fields (B-modes)
- No other sources for a curl polarization field of the CMB at large angular scales:
- B-modes are a clear signature of inflation.







What can we learn from the CMB polarization?

- Polarization measurements
 - Thomson scattering produces LINE/ polarization
 - After scattering, only the quadrupolar component remains



Different polarization modes produced by distinct processes

 \checkmark scalar \rightarrow density perturbations

 \checkmark Vector \rightarrow Vorticity in the primordial plasma (not observed in the standard model)

- \checkmark Tensor \rightarrow primordial gravitational waves deform the potential wells
- THESE ARE DEPENDENT ON THE CHOICE OF GAUGE



Polarization information not contained in the CMB data

- Direct evidence of primordial gravitational waves \Rightarrow B mode, r>0.01, a detection through polarization will be possible
- Reionization and star formation \Rightarrow large angular scales, $\theta > 180/z_{reio} ~ 7^{\circ}$ (E mode only)
- CMB quadrupole when its photons crossed galaxy clusters ⇒ a22(zclust) (E-mde only) (Kamionkowksi & Loeb, PRL 1997)
- $_{\odot}$ Hints of cosmic magnetic fields \Rightarrow circular polarization, V
- Mixing of E and B modes through weak lensing (parity break) ⇒ limits in neutrino masses (Kaplinghat et al., PRL 2003)

THE CMB SECONDARY FLUCTUATIONS

The Sunyaev-Zeldovich Effect

- It occurs when Cosmic Microwave Background (CMB) photons pass through intracluster regions where the gas reaches temperatures T ~ 10⁷ – 10⁸ K.
- There is no reduction in photon number during the cluster crossing, but a shift to higher energies.
- 1 in 100 CMB photons is scattered by electrons in the hot intracluster gas, via inverse Compton scattering.
- Scattering distorts the CMB blackbody spectrum and is independent from the distance to the cluster.



Rachid SUNYAEV



Yakob B. ZEL'DOVICH

Weak lensing of the CMB



Last scattering surface

Inhomogeneous universe - photons deflected



Observer



Slide from A. Lewis



Why is lensing important?

- Act as a CMB contaminant, mainly changing primordial polarization
- Can also account for:
 - non-gaussianity (introduced by lensing distribution) ⇒
 higher order statistics
 - statistical anisotropy (introduced by fixed distribution of lenses) \Rightarrow off-diagonal elements in the covariance matrix
 - Change the CMB power spectrum at large !!!!
- $_{\texttt{O}}$ Useful to break parameter degeneracies and place better constraints on ω and ν energy budget



CMB Anomalies

- Inadequate statistical interretation, according to WMAP team (Bennett et al., ApJ 2011)
- Anomalies were reported as REAL EFFECTS, according to the rest of the communityr (Bernui, Abramo, Andrade, Wuensche, Vielva, Luminet, Eriksen, Huterer, Paselski, Land, Magueijo...)
- Anomalies are real effects, but to a very small level of statistical significance (Planck 2018 papers)
- Interpretation of an "a priori likelihood", specially when the detection of a given event is optimized by the choice of analysis method is not "unbiased". Recurring, complicated and hard to quantify (check Planck 2018 papers:
 - VII. Isotropy and statistics of the CMB
 - IX. Constraints on primordial non-Gaussianity



Non-gaussianity

- ACDM does not predict NG effects
- WMAP estimator: the bispectrum
- $<\Phi_{k_1}, \Phi_{k_2}, \Phi_{k_3}>= (2\pi)^3 \delta^D(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) F(\vec{k}_1, \vec{k}_2, \vec{k}_3)$
- $F=F(P_{\Phi}, f_{NL}, k)$ are "local", "equilateral" and "orthogonal"
- WMAP result: -306 < f_{NL} < + 166 (V+W, foreground marg.)
 consistent with null hypothesis
- Planck should limit |f_{NL}| < 3</p>

WMAP - Bennett et al ApJ, 2011

Planck 2018: IX. Constraints on primordial non-Gaussianity



What can still be done after Planck, SPT and ACT?

- Measure the intrinsic anisotropy to determine the high-l tail of the primary anisotropy and to search for intrinsic non-Gaussianity.
- Measure the gravitational lensing of the CMB. The CMB lensing field can be described by the detection field as T_{obs}(n) = T_{int}(n + d) where T_{int} is the unlensed temperature field, n is the direction, and d is the detection field.
- Find clusters of galaxies through their SZ effect, determine the cluster redshifts with optical follow up, understand the mass selection function with a combination of SZ, optical, and X-ray measurements, and from the cluster catalog determine dN=dz or dN(> M)=dM [3].
- Correlate and compare the CMB with lower redshift cosmological measurements. In the most straightforward application, one uses the interaction of the CMB with lower redshift phenomena and from that determines the growth rate of structure.



Final remarks

Polarization

 large scale B-mode polarization from primordial gravitational waves:

- energy scale of inflation
- rule out most ekpyrotic and pure curvaton/ inhomogeneous reheating models and others
- Small scale B-modes
 - Strong signal from any vector vorticity modes, strong magnetic fields, topological defects

• Weak lensing of CMB

- B-modes potentially confuse primordial signals
- Important correction to theoretical linear result
- The Sunyaev-Zeldovich Effect
 - Excellent tracer of mass distribution at high redshifts
 - Can be used to place strong constraints in Ω_M , Ω_Λ , σ_8



Final remarks

- CMB studies are one of the most interesting and rich fields of study to probe the early Universe. Precision cosmology: a CMB highlight, constraining many cosmological parameters and primordial perturbations
- Deviations from gaussianity: small deviations from standard, near scale-invariant, adiabatic primordial spectrum.
 - Anomalies and power asymmetries are present in the current CMB (space, balloon and ground-based) data
- Polarization: E-mode and T-E cross-correlation optical depth, constrain reionization and isocurvature modes
 - B-modes potentially confuse primordial signals
 - Important correction to theoretical linear results

B-mode polarization is the target for this decade, with ground instruments ongoing and under construction (BICEP, ACT, SPT, Simons Array, Simons Observatory) and a new ESA CoP for a new space mission. CMBPol/COrE is the main candidate



End of Lecture 8



$Q \rightarrow -Q, U \rightarrow -U$ under 90 degree rotation $Q \rightarrow U, U \rightarrow -Q$ under 45 degree rotation

• Note that Stokes parameters refer explicitely to a given coordinate system, so $Q(\theta, \phi)$ and $U(\theta, \phi)$ maps in the sky depend **upon the choice of the coordinate system**. Under a rotation around the z-axis:

$$Q' = Q\cos(2\varphi) + U\sin(2\varphi)$$
$$U' = -Q\sin(2\varphi) + U\cos(2\varphi)$$



From Q, U to E, B

$$Q_{f}(\hat{z}) = \frac{3\sigma_{T}}{16\pi} \int d\Omega \sin^{2}\theta \cos(2\varphi) I_{i}(\theta,\varphi) U_{f}(\hat{z}) = -\frac{3\sigma_{T}}{16\pi} \int d\Omega \sin^{2}\theta \sin(2\varphi) I_{i}(\theta,\varphi) \right\} Q - iU \sim \int d\Omega Y_{22}(\theta,\varphi) I(\theta,\varphi)$$

Polarization depends upon the quadrupole of the incident radiation. It is convenient to write U, Q in terms of the complex function P:

$$P = Q - iU = \frac{3\sigma_T}{4\pi} \sqrt{\frac{2\pi}{15}} a_{22}$$

 $\Pi = E + iB$

NOT invariant under rotation... We need to construct a new function, through quantum lowering operators, that is invariant under rotation!



E (or G) is the gradient mode

 → zero rotational
 → even under reflextion

B (or C) is the rotational mode zero gradiente odd under reflexion

- Expand scalar P_E and P_B in spherical harmonics
- Expand P_{ab} in tensor spherical harmonics

$$E_{lm} = \sqrt{2} \int_{4\pi} \mathrm{d}S \, Y^{G\,ab*}_{(lm)} \mathcal{P}_{ab} \qquad \qquad B_{lm} = \sqrt{2} \int_{4\pi} \mathrm{d}S \, Y^{C\,ab*}_{(lm)} \mathcal{P}_{ab}$$

$$\mathcal{P}_{ab} = \frac{1}{\sqrt{2}} \sum_{lm} \left(E_{lm} Y^G_{(lm)ab} + B_{lm} Y^C_{(lm)ab} \right)$$

Figures by Wayne Hu (http://background.uchicago.edu/~whu/intermediate/Polarization/







From Paolo de Bernardis (lecture @ INPE's IV Advanced School)



From Paolo de Bernardis (lecture @ INPE's IV Advanced School)



Primary/secondary sources of polarization: free e-, with a given optical depth for Thomson scattering:

z < 1100 weak lensing from Large Scale Structure

> z ~ 6-30 (?) Reionization

z < 3 Clusters (SZ)

z ~ 1100 @ LSS

Magnetic field (Faraday rotation)

Slide from Raul Abramo

z~0

Galaxy



Primordial B modes

 The amplitude of this effect is very small, but depends on the Energy scale of inflation. In fact the amplitude of tensor modes normalized to the scalar ones is:

$$R = \left(\frac{T}{S}\right)^{1/4} \equiv \left(\frac{C_2^{GW}}{C_2^{Scalar}}\right)^{1/4} \cong \frac{V^{1/4}}{3.7 \times 10^{16} \text{ GeV}}$$

• and
$$\sqrt{\frac{\ell(\ell+1)}{2\pi}} c_{\ell \max}^B \cong 0.1 \mu K \left[\frac{V^{1/4}}{2 \times 10^{16} \text{ GeV}}\right]$$

There are theoretical arguments to expect that the energy scale of inflation is close to the scale of GUT i.e. around 10¹⁶ GeV.
 The measurement of B-modes is a good way to investigate fundamental physics at extremely high energies.



B-modes and lensing of E-modes

- E-modes have been measured already with good accuracy, and will be measured with exquisite accuracy by Planck and other experiments.
 - They depend on the distribution of mass (mainly dark matter) so their study can shed light on the nature of dark matter (including massive neutrinos).
 - While the primordial B-mode is maximum at multipoles around 100 (θ =2°), the lensed B-mode is maximum at multipoles around 1000 (θ =0.2°), requiring high angular resolution polarization experiments





Chiang et al. 2010



From Paolo de Bernardis (lecture @ INPE's IV Advanced School)



Differencing occurs here

OMT splits the signal into 2 polarizations

A+B'/sqrt(2) and A-B'/sqrt(2)





Primordial Perturbations: fluid at redshift < 10⁹

- Photons
- Nearly massless neutrinos

Free-streaming (no scattering) after neutrino decoupling at z 10⁹

Baryons + electrons

tightly coupled to photons by Thomson scattering

Dark Matter

Assume cold. Coupled only via gravity.

Dark energy
 probably negligible early on



Tests of Lambda-CDM and non-gaussianity



 The 26 most significant clusters over full 2500 deg2 SPT survey.
 Tests extreme tail of matter power spectrum (high-mass, highredshift clusters)

– Even a single massive cluster could indicate tension with λ CDM (Mortonson, Hu, Huterer 2010).

We find:

- consistent with λCDM
- consistent with initial Gaussian density fluctuations

slide from John Carlstrom Cluster Gas Pressure is a "Clean" Mass Estimator







l
$$\begin{split} \widehat{U} & Lensing \ \text{Potential} \\ \widetilde{T}(\vec{x}) = T(\vec{x} + \alpha(\vec{x})) \\ [\widetilde{Q} \pm i\widetilde{U}](\vec{x}) = [\widetilde{Q} \pm i\widetilde{U}](\vec{x} + \alpha(\vec{x})) & \text{Field of deflection vectors} \\ \alpha(\vec{x})) = -2 \int_{0}^{\chi_{*}} d\chi \frac{\chi_{*} - \chi}{\chi_{*}} \nabla_{\perp} \Phi(\chi \vec{x}; \eta_{0} - \chi) \end{split}$$

 LSS can be thought of as a collection of potential wells acting as point sources for lensing
Deflections O(10⁻⁴), or 2', but coherent on degree scales -> important!





Lensing potential and deflection angles



LensPix sky simulation code: http://cosmologist.info/lenspix

Lensed CMB power spectra



Slide from A. Lewis



Lensing of CMB polarization



Hanson, Challinor, Lewis, astro-ph:0911.0612

Nearly white BB spectrum on large scales Potential confusion with tensor modes Leakage from E to B-modes Lensing effect can be largely subtracted if only scalar modes + lensing present, but approximate and complicated (especially posterior statistics).

> Hirata, Seljak : astro-ph/0306354 Okamoto, Hu: astro-ph/0301031

INPE

WEAK-LENSING MASS MEASUREMENTS OF FIVE GALAXY CLUSTERS IN THE SOUTH POLE TELESCOPE SURVEY USING MAGELLAN/MEGACAM



FIG. 16.— SZ, optical, and κ data for SPT-CL J2135-5726. See Section A for a description.

















/ 14.6°





J 41.6°





Limitations?

- Foregrounds understanding limits:
 - Polarization measurements => synchrotron and dust emission can be > 70% polarized
 - SZ measurements ⇒ limited by our ability to measure the X-ray flux of extragalactic radio sources and by distortions caused by lensing
 - CMB lensing => can be limited by SZ and the residual radio background
- CMB lensing and SZ can hamper each other measurements



Science with Planck

- Determine the large scale properties of the Universe with highest precision
- Test inflation and search for primordial gravitational waves
- Search for defects in the space-time and non-gaussian signatures in the otherwise Gaussian CMB distribution
- Study the origin of present-day large scale structures
- Study our Galaxy (and other galaxies) in microwave and submillimeter



Planck results (no CMB)



Clumps corresponding to dusty galaxies (in large scales and throughout cosmic history) Measurements of the CIB



Measurements in Rho Ophiuchus: correlation between anomalous emission in µ-waves (30 GHz) and thermal dust emission (857 GHz)



Source: Planck collaboration Discovery of the first supercluster of galaxies, (PLCK G214.6+37.0) detected by Planck and confirmed by XMM-Newton.

