

## WINDS, B-FIELDS, AND MAGNETOTAILS OF PULSARS

M. M. ROMANOVA, G. A. CHULSKY, AND R. V. E. LOVELACE

Cornell University, Ithaca, NY 14853

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## ABSTRACT

We investigate the emission of rotating magnetized neutron stars due to the acceleration and radiation of particles in the relativistic wind and in the magnetotail of the star. We consider that the charged particles are accelerated by driven collisionless reconnection. Outside of the light cylinder, the star's rotation acts to wind up the magnetic field to form a predominantly azimuthal, slowly decreasing with distance, magnetic field of opposite polarity on either side of the equatorial plane normal to the star's rotation axis. The magnetic field annihilates across the equatorial plane with the magnetic energy going to accelerate the charged particles to relativistic energies. For a typical supersonically moving pulsar, the star's wind extends outward to the standoff distance with the interstellar medium. At larger distances, the power output of pulsar's wind  $\dot{E}_w$  of electromagnetic field and relativistic particles is *redirected and collimated into the magnetotail* of the star. In the magnetotail it is proposed that equipartition is reached between the magnetic energy and the relativistic particle energy. For such conditions, synchrotron radiation from the magnetotails may be a significant fraction of  $\dot{E}_w$  for high velocity pulsars. An equation is derived for the radius of the magnetotail  $r_m(z')$  as a function of distance  $z'$  from the star. For large distances  $z'$ , of the order of the distance travelled by the star, we argue that the magnetotail has a 'trumpet' shape owing to the slowing down of the magnetotail flow. We compare results with the Guitar and Mouse nebula, and conclude that the tail of the Mouse may be connected with the long magnetotail behind the pulsar. We argue that the shock waves and elongated structures may also be observed in a misdirected or shutoff pulsars and may be used as a tool for finding these objects.

*Subject headings:* stars: neutron — pulsars: general — stars: magnetic fields — X-rays: stars

## 1. INTRODUCTION

The relativistic winds of supersonically moving pulsars are observed to form shock waves in the interstellar medium (ISM) and in some cases prominent wakes or "tails". Several pulsars are known to power bow shock waves and tails, the "Guitar" nebula (Cordes et al. 1993; Chatterjee & Cordes 2002), millisecond pulsars (Kulkarni & Hester 1988; Bell et al. 1995), the "Duck" nebula (e.g., Thorsett, Briskin, & Goss 2002), the "Mouse" nebula, observed in the X-ray and radio bands and powered by the spin-down of a 98 ms pulsar (Camilo et al. 2002; Gaensler et al. 2004), the pulsar B0740-28 (Jones, Stappers, & Gaensler 2002), and the neutron star candidate RX J1956.5-3754 (van Kerkwijk & Kulkarni 2001a, b). Axisymmetric simulations of the interaction of magnetized stars with the interstellar medium (ISM) have shown that such interaction leads to formation of long magnetotails behind the stars (Romanova et al. 2001; Toropina et al. 2001). Such magnetotails may allow relativistic particles from the pulsar to propagate to large distance behind the pulsar.

In isolated pulsars a relativistic magnetohydrodynamic (MHD) wind forms outside of the light cylinder and propagates outward (Goldreich & Julian 1969, hereafter GJ69; Arons & Tavani 1994; Arons 2004). This wind forms a magnetically dominated, azimuthally wrapped disk-like equatorial structure which expands with velocities of the order of the speed of light and is driven mostly by the magnetic pressure. Observations reveal disk-like structures around a number of pulsars, including one in the Crab nebula (Weisskopf, et al. 2000), the Vela pul-

sar (Pavlov et al. 2001), and a few other cases (Helfand, Gotthelf, & Halpern 2001; Gotthelf 2001). The orientation and thickness of these disk-like structures (or, PWN tori) were estimated by Ng and Romani (2004) who suggested that particles are accelerated more efficiently in these disk-like structures. It was also noticed that the rotation axis of the disk approximately coincides with the direction of pulsar's velocity, so that the geometry of the flow is approximately axi-symmetric (Spruit & Phinney 1998; Lai, Chernoff, & Cordes 2001). In a few cases jets were observed which are approximately aligned with the direction of motion (e.g., Weisskopf, et al. 2000).

Formation of such an equatorial disk structure was recently modelled in axisymmetric relativistic MHD simulations by Del Zanna, Amato, and Bucciantini (2004), and Komissarov and Lyubarsky (2004) for the case of an initial split monopole magnetic field. They have shown that a magnetically-dominated disk is formed around a rotating pulsar and that this disk has a small thickness. Formation of rotating, azimuthally wrapped disk-like structures were also observed in non-relativistic axisymmetric simulations of an accreting, rotating star with an aligned dipole magnetic field in the propeller regime (Romanova et al. 2003). This disk-like magnetically dominated bulk flow carries most of energy out of the pulsar, however, it may be invisible unless the particles are accelerated by some mechanism.

Different models have been proposed to explain acceleration of particles in this wind. In one class of models *ideal* relativistic magnetohydrodynamics is assumed to hold outside of the light cylinder with the acceleration of plasma due to the spatial variation of the electric and

magnetic fields (e.g., Vlahakis 2004). In these models it is suggested that the Poynting-flux to matter-energy-flux ratio ( $\sigma$ ) evolves from a value much larger than unity near the light cylinder to a value less than unity at large distances where the pulsar wind encounters the interstellar medium. In another class of models *non-ideal* plasma effects are considered to be responsible for the particle acceleration. The particle acceleration may be stochastic as discussed in the shock wave acceleration models (Arons & Tavani 1994; Arons 2004; Spitkovsky & Arons 2004). Alternatively in collisionless reconnection and annihilation the magnetic field may cause random (Coroniti 1990) or bulk acceleration of the particles (Lyubarsky & Kirk 2001; Kirk & Lyubarsky 2001). These models argue that the MHD wind consists of an azimuthally wrapped dipole magnetic field of the star with regions where the magnetic field reverses direction. These regions are likely sites for reconnection and particle acceleration.

In this work we investigate a model where particles are accelerated in the pulsar's *equatorial* neutral layer across which the predominantly azimuthal magnetic field reverses direction. There may also be oppositely directed Poynting flux flows aligned with the star's rotation axis analogous to those generated from the time-dependent expansion of magnetic field loops of an accretion disk into a low density coronal plasma discussed by Lovelace and Romanova (2003) and simulated by Lovelace, Gandhi, and Romanova (2004) with a relativistic electromagnetic particle-in-cell method. However, note that numerical solutions of the "pulsar equation" for an aligned stationary magnetosphere by Contopoulos, Kazanas, and Fendt (1999) and Gruzinov (2005) do not show Poynting flows along the rotation axis.

Our model is qualitative in that we do not have global self-consistent calculations of the electromagnetic fields and sources. Compared to Coroniti (1990) we suppose that the misalignment angle between the rotational and magnetic axes is *not large*. Then, instead of multiple neutral layers in a system of magnetic stripes, a global extended neutral layer forms in the equatorial plane. The equatorial layer is not expected to be thin everywhere. However, the systematic acceleration will be in the radial direction. Thus the bulk motion of plasma and magnetic field will be in the radial direction mainly in the equatorial plane. Thus, we expect to have a high- $\sigma$  plasma near the light cylinder which gradually changes to a much lower  $\sigma$  far from the pulsar. Accelerated particles will gradually diffuse vertically away from the thin neutral layer, and their interaction with the magnetic field will produce the synchrotron radiation. In this paper we develop this model in semi-quantitative detail.

The acceleration of particles in collisionless reconnection has been discussed by a number of authors (Alfvén 1968; Dessler 1969, 1971; Speiser 1970; Cowley 1971, 1973; Bulanov & Sasorov 1976; and Vasylunas 1980; Burkhart, Drake, & Chen 1991). The present work uses the model by Alfvén (1968) and its relativistic counterpart (Romanova & Lovelace 1992; Zenitani & Hoshino 2001; Larrabee, Lovelace, & Romanova 2003). Both the simulations (Zenitani & Hoshino 2001) and the analytic theory (Larrabee et al. 2003) indicate a power law distribution of accelerated particles.

Most pulsars propagate through the ISM highly supersonically so that the pulsar wind forms a strong shock

wave in the ISM. A model of this interaction was first discussed by Schwartzman (1970). Subsequently, the shape of the shock wave was calculated for the hydrodynamic flow by Baranov, Krasnobaev & Kulikovsky (1971), Baranov & Malama (1993) and by Wilkin (1996). However, for the pulsar case it is necessary to include the relativistic magnetized wind of the star. Recent models of the Solar wind interaction with the ISM include the influence of the magnetic field for both the interstellar and interplanetary components, and these models show that the shape of the bow shock may depend on these fields (e.g., Linde et al. 1998; Zank 1999; Bucciantini et al. 2004; Opher et al. 2004). However, only relatively low Mach numbers are considered and the Alfvén speeds are typically small compared with the flow speeds. In the case of pulsars the Mach numbers are much larger than unity and the Alfvén speeds may be comparable with the flow speeds. Strongly magnetized magnetotails were observed in axisymmetric simulations of rotating and non-rotating strongly magnetized stars (Toropina et al. 2001).

In this paper we discuss the nature of the magnetotails which form behind the pulsars. We argue that most of the power of the star's wind of relativistic particles and magnetic field is *redirected* by the internal shock wave and subsequently propagates down the magnetotail. In several cases observations show cometary structures aligned with the direction of propagation of the pulsar (e.g., Stappers et al. 2003; Gaensler et al. 2003) which may be the result of the pulsar wind propagating down the magnetotail.

Both shocks and magnetotails may appear in the case of misdirected pulsars or longer-period, shutoff pulsars. We discuss the possible nature of the isolated neutron star candidate RX J1956.5-3754 which has a shock wave.

In §2 we discuss basic considerations. In §3 we discuss particle acceleration due to reconnection in the equatorial neutral layer of the star, and in §4 the associated synchrotron radiation. In §5 we discuss the bow shock wave and the related magnetotail and its synchrotron emission. In §6 we discuss examples of pulsars with tails. In §7 we consider the possibility of observing wind and magnetotail emitting stars in the Solar neighborhood. In §8 we give the conclusions of this work.

## 2. BASIC CONSIDERATIONS

A pulsar's light cylinder radius is

$$r_L = \frac{c}{\omega_*} = \frac{Pc}{2\pi} \approx 4.8 \times 10^9 P \text{ cm}, \quad (1)$$

where  $P$  is the period in seconds. The pulsar spins down owing to the relativistic wind which flows outward beyond the light cylinder distance (GJ69). This wind consists of an electromagnetic field and relativistic particles and has a power output

$$\dot{E}_w \approx B_L^2 r_L^2 c = \frac{\mu^2 \omega_*^4}{c^3} \approx 5.8 \times 10^{31} \frac{\mu_{30}^2}{P^4} \frac{\text{erg}}{\text{s}}, \quad (2)$$

(GJ69), where  $B_L \equiv \mu/r_L^3$  is the magnetic field strength at the light cylinder radius, and  $\mu_{30} = \mu/(10^{30} \text{ Gcm}^3)$  is the star's magnetic moment. The star's surface magnetic field is  $B = B_{12} 10^{12} \text{ G}$ , with  $B_{12} = \mu_{30}/R_6$ , where  $R_6 = R_*/10^6 \text{ cm}$  and  $R_*$  the star's radius.

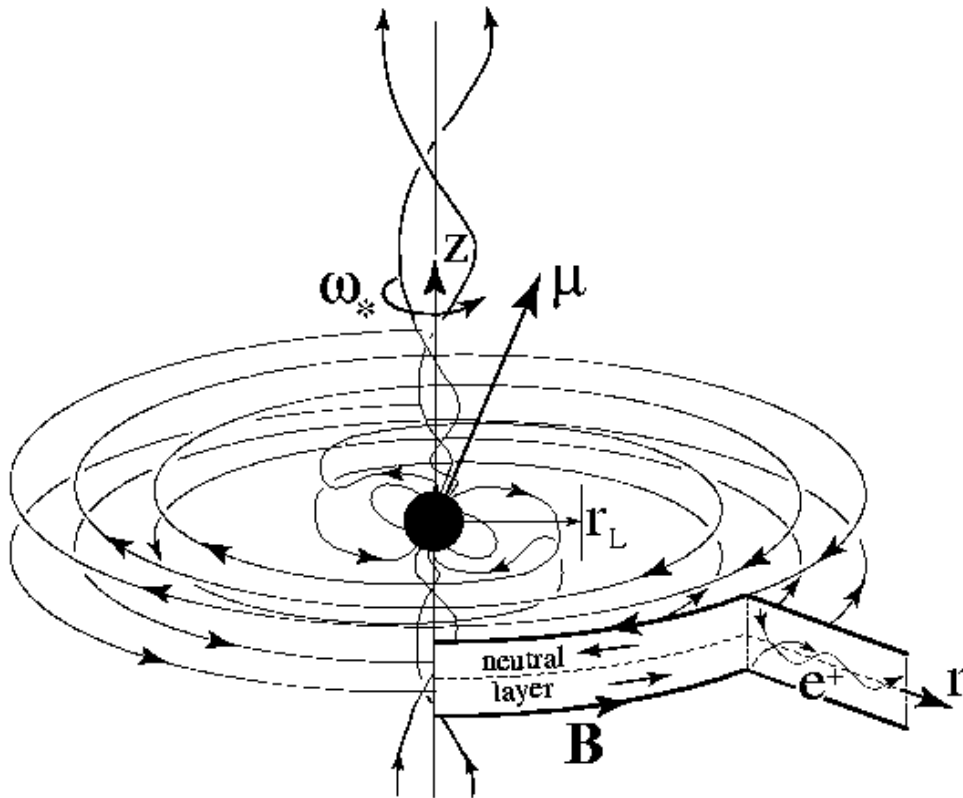


FIG. 1.— Sketch of the magnetic field configuration of misaligned rotating neutron star outside of the light cylinder,  $r > r_L = c/\omega_*$ . For the case shown  $\Omega \cdot \mu > 0$  the magnetic field is wrapped in clock-wise spiral in the upper half-space and a counter-clockwise spiral in the lower half-space. A neutral layer is formed in the equatorial plane where there is forced reconnection or annihilation of the magnetic field. The magnetic field energy goes into accelerating particles to relativistic energies. The helical field lines around the  $\pm z$  axes correspond to possible Poynting outflows.

Assuming the pulsar wind approximately isotropic, the pressure of the wind varies as  $p_w = \dot{E}_w / (4\pi R^2 c)$  for distances  $R > r_L$  from the star. For a pulsar moving supersonically through the ISM, the pressure of the pulsar wind  $p_w(z)$  balances the ram pressure of the ISM  $\rho_{ism} v^2$  at the stagnation point at a distance  $z_{sh}$  in front of the pulsar. This is the location of the bow shock of the pulsar. That is,  $p_w(z_{sh}) = \rho_{ism} v^2$  so that

$$z_{sh} = \frac{B_L r_L}{(4\pi \rho_{ism} v^2)^{1/2}} \approx 10^{14} \frac{B_{12}}{n_{ism}^{1/2} v_8 P^2} \text{ cm}, \quad (3)$$

(Schvartsman 1970; Baranov et al. 1971; Wilkin 1996), where  $\rho_{ism}/m_p \approx n_{ism}$  (in units of  $\text{cm}^{-3}$ ) is the number density of the ISM with  $m_p$  the proton mass. The lateral width of the bow shock is  $\approx z_{sh}$ . Thus the power which goes into *thermal heating* of the shocked interstellar gas is

$$\dot{E}_{sh} \approx \frac{3}{16} \rho_{ism} v^3 \pi z_{sh}^2 \approx 0.9 \times 10^{28} \frac{v_8 B_{12}^2}{P^4} \frac{\text{erg}}{\text{s}}, \quad (4)$$

where the  $3/16$  factor assumes a specific heat ratio of  $5/3$ . The power  $\dot{E}_{sh}$  is extracted from the kinetic energy of the motion of the star through the ISM. The slowing of the star's motion is however negligible. Notice that the ratio

$$\frac{\dot{E}_{sh}}{\dot{E}_w} \approx \frac{3}{64} \frac{v}{c} \approx 1.56 \times 10^{-4} v_8, \quad (5)$$

is much smaller than unity.

### 3. MAGNETIC RECONNECTION

Outside of the light cylinder the pulsar's magnetic field is predominantly azimuthal. If the magnetic moment of the star is perpendicular to its rotation axis, then the azimuthal magnetic field forms “stripes” of opposite polarity which move outward relativistically (e.g., Michel 1971, Coroniti 1990, Lyubarsky & Kirk 2001). Particles may be gradually accelerated by reconnection of the oppositely directed fields (Coroniti 1990). Most of the power flow is thought to be in the form of a magnetically dominated wind. That is, the ratio of the magnetic energy density to the kinetic energy density of the particles, the  $\sigma$  parameter, is much larger than unity. As the distance from the light cylinder increases,  $\sigma$  decreases.

We consider the case where the angle between the magnetic and rotation axes is not very large. The nature of the envisioned driving mechanism is sketched in Figures 1 and 2. This work focuses on the equatorial region of the flow, but for completeness the figures show possible Poynting outflows along the  $\pm z$  axes. The plasma behavior is *non-ideal*; it is *not* described by ideal MHD. We use both cylindrical  $(r, \phi, z)$  and spherical  $(R, \theta, \phi)$  non-rotating coordinates to describe the electromagnetic field and particle motion for  $R > r_L$ . The rotation of the star wraps the field around the rotation axis so that it is

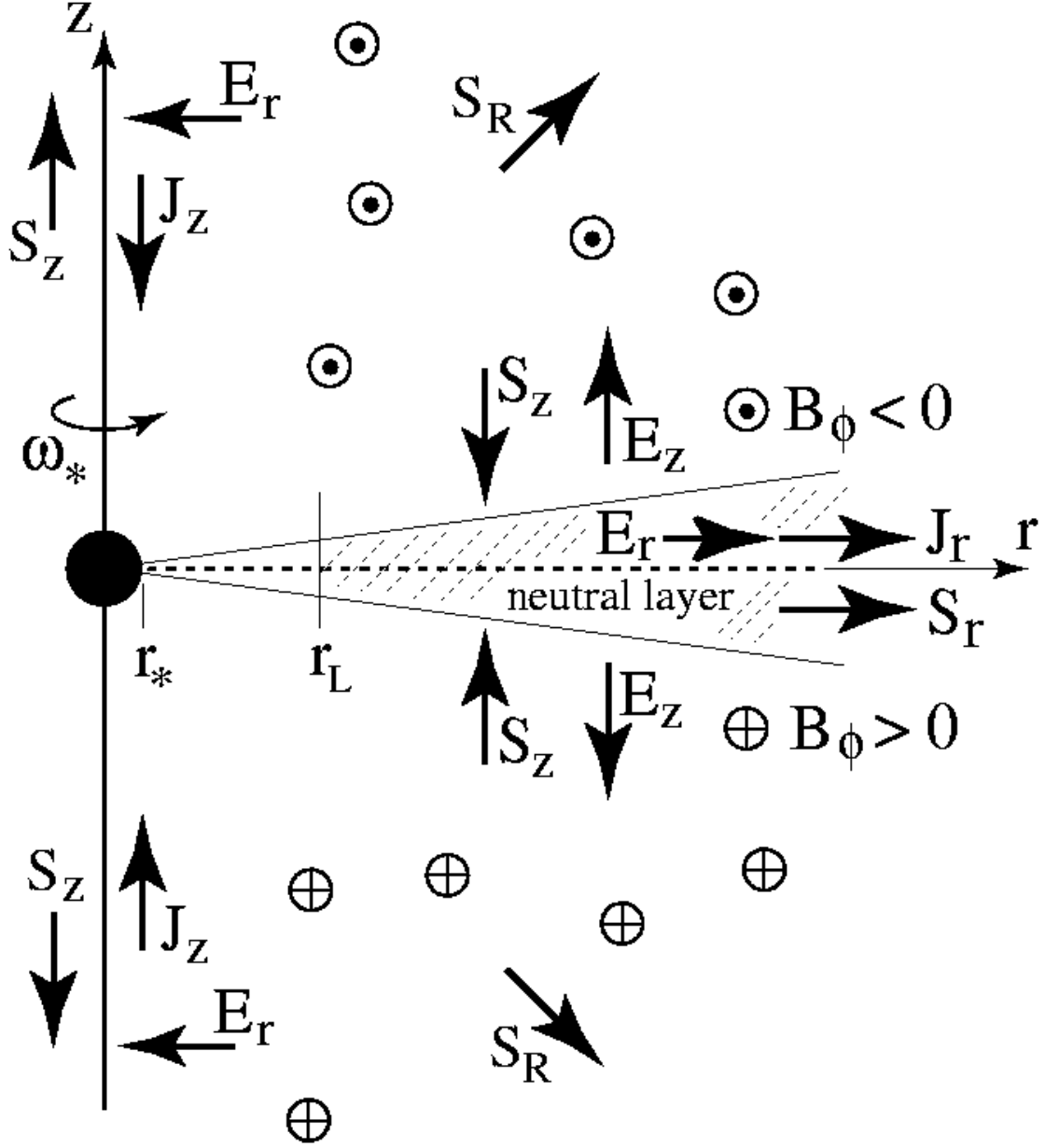


FIG. 2.— The figure shows the current flows ( $\mathbf{J}$ ), electric fields ( $\mathbf{E}$ ), and Poynting vectors ( $\mathbf{S} = c\mathbf{E} \times \mathbf{B}/4\pi$ ) of the envisioned configuration outside of the light cylinder,  $r > r_L = c/\omega_*$ .

predominantly azimuthal as shown in Figure 1. For the case assumed here,  $\omega_* \cdot \boldsymbol{\mu} > 0$ , the field lines are wound up clockwise for  $z > 0$  and counter-clockwise for  $z < 0$ . Consequently, there is a *neutral layer* in equatorial plane where the azimuthal magnetic field changes from  $B_\phi > 0$  for  $z < 0$  to  $B_\phi < 0$  for  $z > 0$ . Within the neutral layer, the magnetic field is approximated as

$$B_r = B_L \left( \frac{r_L}{r} \right)^2 \left( \frac{z}{\Delta z} \right), \quad B_\phi = -B_L \left( \frac{r_L}{r} \right) \left( \frac{z}{\Delta z} \right), \quad (6)$$

where  $\Delta z(r)$  is the half-thickness of the neutral layer

which is assumed less than  $r$ , and where  $B_L \equiv \mu/r_L^3 \approx 9.2 \mu_{30}/P^3$  G. Thus the magnetic field has form of Archimedes' spiral with field lines given by  $r = \text{const} - r_L \phi$ . Equation (6) neglects the effect of the annihilation of the field.

In the neutral layer Ampère's law gives  $\partial B_\phi / \partial z = -4\pi J_r / c$ . Thus the change in the azimuthal field across the layer is

$$\Delta B_\phi = \left[ B_\phi \right]_{-\Delta z}^{\Delta z} = -\frac{2I_r}{c r}. \quad (7)$$

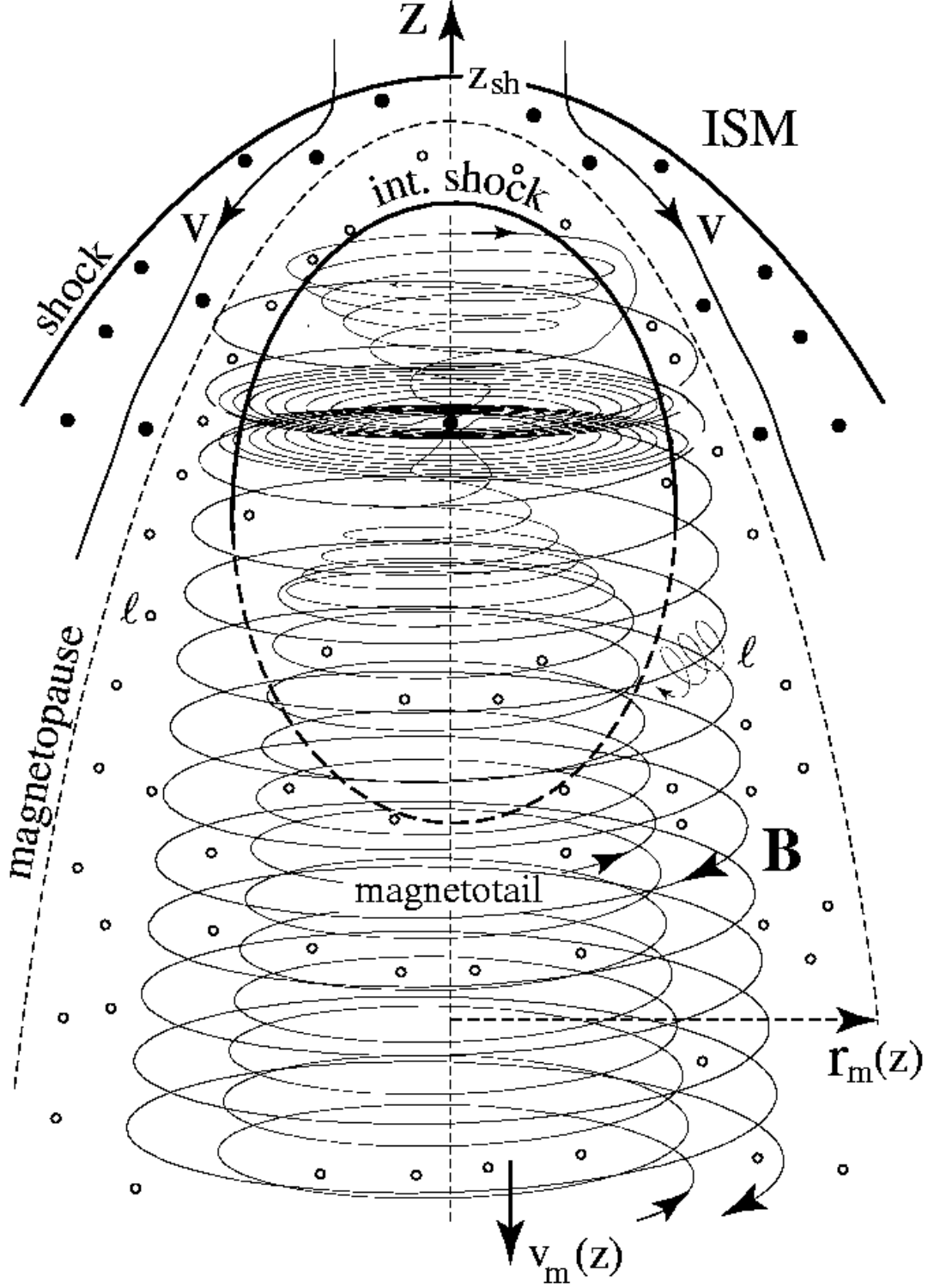


FIG. 3.— Sketch of the external shock, the magnetopause, and the magnetotail of a pulsar moving supersonically through the interstellar medium. The velocity of the star is assumed parallel to the star's rotation axis  $\omega_*$ . The solid circles indicate the shocked interstellar gas with sample streamlines labelled by  $\mathbf{v}$ . The open circles indicate relativistic leptons ( $\ell$ ) which flow into the magnetotail. The radius of the magnetotail is  $r_m(z)$  and the bulk velocity of the magnetotail plasma is  $\mathbf{v}_m(z)$ . The internal shock of the pulsar wind is indicated by “int. shock.” The standoff distance of the shock,  $z_{sh}$ , is given by equation (3).

Thus, the radial current carried by the neutral layer is

$$I_r = B_L r_L c \approx 4.4 \times 10^{11} \frac{\mu_{30}}{P^2} \text{ A} , \quad (8)$$

where  $\mu_{30} \equiv \mu/(10^{30} \text{ G cm}^3)$  is the magnetic moment of the star. This radial equatorial current flow acts to

*magnetically pinch* the particles within the neutral layer. An analogous current layer exists in the solar wind (e.g., Bertin & Coppi 1985).

The equatorial outflow of current is balanced by an axial inflow of current as indicated in Figure 2. This current corresponds to a particle outflow rate of

$$\frac{dN}{dt} \approx \frac{I_r}{e} \approx 2.7 \times 10^{30} \frac{\mu_{30}}{P^2} \text{ s}^{-1}. \quad (9)$$

There may be oppositely directed Poynting flux outflows along the rotation axis with  $S_z = (c/4\pi)E_r B_\phi \gtrless 0$  for  $z \gtrless 0$  flow outward along the star's rotation axis. These outflows would be analogous to the Poynting outflows from accretion disks (Lovelace & Romanova 2003; Lovelace et al. 2005).

For the considered case  $\mathbf{\Omega} \cdot \boldsymbol{\mu} > 0$ , the electric charge of the neutral layer is dominantly positive. Thus the charge of the layer gives rise to an outward radial electric field  $E_r > 0$  and an axial field  $E_z > 0$  for  $z > 0$  and  $E_z < 0$  for  $z < 0$  as shown in Figure 2. The electric field gives rise to an axial  $\mathbf{E} \times \mathbf{B} = E_r B_\phi \hat{\mathbf{z}}$  ( $< 0$  for  $z > 0$  and  $> 0$  for  $z < 0$ ) drift of the azimuthal field and associated plasma *into* the neutral layer. This constitutes the driving force for the reconnection.

Additionally, there is a radially outward  $\mathbf{E} \times \mathbf{B} = -E_z B_\phi \hat{\mathbf{r}}$  drift of the particles in the neutral layer. The  $\mathbf{E} \times \mathbf{B}$  drifts are of course in the same direction as the Poynting vector  $\mathbf{S} = (c/4\pi)\mathbf{E} \times \mathbf{B}$  which is shown in Figure 2.

Particles are accelerated in the electric field  $E_r$  which decreases with the distance as  $1/r$  or faster. This dependence is determined by the fact that the energy comes from the magnetic field,  $\sim B_\phi$ , which varies as  $1/r$  (see equation 6). There is an additional factor in equation (6),  $z/\Delta z$ , which takes into account the fact that at larger distances the relative thickness of the neutral layer will likely increase due to instabilities and turbulence. The instabilities may be driven by the misalignment of the rotation  $\mathbf{\Omega}$  and magnetic  $\boldsymbol{\mu}$  axes. This is expected to lead to faster than  $1/r$  decrease of the field with the distance. In addition, part of the energy of the field will go to acceleration of particles, which will cause  $E_r$  to fall off more rapidly than  $1/r$ . Thus, we introduce a parameterized dependence of the electric field within the neutral layer in the form

$$E_r = \alpha_E B_L \left( \frac{r_L}{r} \right)^q, \quad (10)$$

where  $\alpha_E$  is a dimensionless quantity assumed to be a constant less than unity, and  $q$  is another constant which characterizes how fast this electric field decreases with the distance. The overall electrical neutrality of the equatorial and axial regions implies that  $q > 1$ . At the boundary of the neutral layer  $E_z(r, \Delta z) \approx \alpha'_E B_L (r_L/r)$ , where  $\alpha'_E$  is further dimensionless quantity assumed to be appreciably less than unity.

Neglecting for the moment radiative energy losses, positively charged particles drifting into the neutral layer are accelerated in the direction  $\hat{\mathbf{r}} + (r_L/r)\hat{\boldsymbol{\phi}}$  which is approximately radial for  $r \gg r_L$ . The radial acceleration gives

$$\frac{d}{dt}(mc^2\gamma) \approx c \frac{d}{dr}(mc^2\gamma) \approx q\alpha_E c B_L \left( \frac{r_L}{r} \right)^q, \quad (11)$$

where  $m$  is the particle rest mass and  $e$  is its charge (Alfvén 1968). The approximation involves assuming that the particles move outward with speed  $\approx c$ . Integrating eq. (11) from  $r = r_L$  to  $r \gg r_L$ , we find

$$\gamma = 1 + \frac{e B_L r_L}{mc^2} \frac{\alpha_E}{(q-1)}, \quad (12)$$

for  $r \gg r_L$  and  $q$  not close to unity.

$$\gamma \approx 2.6 \times 10^7 \left( \frac{\alpha_E}{q-1} \right) \left( \frac{\mu_{30}}{P^2} \right), \quad (13)$$

for the case of leptons. For protons and heavier ions the numerical factor is  $\lesssim 1.4 \times 10^4$ . A smaller number of negatively charged particles is accelerated back towards the pulsar.

The outward kinetic energy flux of the particles accelerated in the neutral layer is

$$\begin{aligned} \dot{E}_{kin} &= mc^2(\gamma-1) \frac{dN}{dt} = (B_L^2 r_L^2 c) \frac{\alpha_E}{q-1} \\ &\approx 5.8 \times 10^{30} \left( \frac{\alpha_E}{0.1} \right) \frac{\mu_{30}^2}{P^4} \frac{\text{erg}}{\text{s}} \end{aligned} \quad (14)$$

for  $r \gg r_L$ , neglecting the synchrotron losses. This kinetic energy flux is a fraction  $\alpha_E/(q-1)$  of the Goldreich-Julian power ( $B_L^2 r_L^2 c$ ).

We expect different behaviors for the case where  $q$  is appreciably larger than unity ( $q-1 \sim 1$ ), and the case when  $q$  is close to unity ( $q-1 \ll 1$ ). In the first case particles are accelerate mainly in the vicinity of the light cylinder. In the second case, they are accelerated along the neutral layer out to much larger distances. The total energy, however, can not be larger than the rotational energy of the pulsar, that is  $\alpha_E/q-1 \leq 1$ .

Particles, accelerated in the neutral layer will be gradually scattered by irregularities (Alfvén waves) in the magnetic field and will drift away from the neutral layer. At the same time there may be magnetic confinement of particle to the neutral layer out to large distances. The force balance (or magnetic pinch) condition for the neutral layer follows from the momentum equation  $\partial T_{zz}/\partial z = 0$  or  $T_{zz} = \text{const}$  (Bertin & Coppi 1985), where  $T_{jk}$  is the stress tensor for the particles and fields. This condition implies that  $\alpha_E = (\Delta z/r)[1 - (\alpha'_E)^2]$ . The compressive pinch force on the neutral layer is responsible for the reconnection at  $z = 0$  being driven.

With increasing distance from the light cylinder the ratio of the Poynting to particle energy fluxes  $\sigma$  decreases. The variation of  $\sigma$  with distance may be anisotropic, with smaller  $\sigma$  in the disk and larger  $\sigma$  above and below the disk. With increasing  $r$ , a larger fraction of the magnetic energy is converted to particle energy in the neutral layer. Particles accelerated in the neutral layer may subsequently scatter so as to give a more spherical wind. The scattering and diffusion processes of particles accelerated in the neutral layer remains to be investigated in detail. For simplicity of the following analysis we suppose that  $\sigma \lesssim 1$  before the bow shock is reached so that the flow is super-Alfvénic and supersonic at the bow shock.

#### 4. SYNCHROTRON RADIATION

The acceleration of leptons in the neutral layer is inhibited due to the energy lost to synchrotron radiation.

Including the synchrotron losses gives

$$\frac{d\gamma}{d\tilde{r}} = \frac{\gamma_0}{\tilde{r}^q} - \frac{k}{\tilde{r}^2} \gamma^2. \quad (15)$$

Here,  $\tilde{r} \equiv r/r_L$ ,  $\gamma_0 \equiv e\alpha_E B_L r_L / (mc^2)$ ,  $k \equiv (2/3)\beta_R B_L^2 r_L r_e^2 / (mc^2)$ , with  $r_e = e^2 / (mc^2)$  the classical electron radius, and  $\beta_R < 1$  accounts for the fact that the magnetic field close to the neutral layer is less than  $B_L / \tilde{r}$ . We assume that  $\gamma \sim 1$  at the light cylinder distance  $\tilde{r} = 1$ . The synchrotron radiation of ions is negligible. Equation (15) represents a simplified model of the synchrotron losses from the neutral layer in that it neglects the fact that the leptons tend to be focused to the  $z = 0$  plane where the magnetic field is appreciably reduced. However, this focusing will be offset if as expected there are significant irregularities (turbulence) in the plasma and the magnetic field as in the Solar wind.

For simplicity we examine the case where  $q = 2$ . It is clear that the Lorentz factors will not exceed

$$\begin{aligned} \gamma_{max} &= \left(\frac{\gamma_0}{k}\right)^{1/2} = \left(\frac{3e\alpha_E r_L^3}{2\beta_R r_e^2 \mu}\right)^{1/2} \\ &\approx 3.1 \times 10^7 \left(\frac{\alpha_E}{\beta_R}\right)^{1/2} \frac{P^{3/2}}{\mu_{30}^{1/2}}. \end{aligned} \quad (16)$$

Thus the synchrotron losses are significant for pulsar periods  $P$  shorter than the period which gives  $\gamma_{max} = \gamma_0$ . This critical period is

$$P_{cr} = \frac{2\pi}{c} \left(\frac{2\alpha_E \beta_R r_L^3 \mu^3}{3emc^2}\right)^{1/7} \approx 0.49 \left(\frac{\alpha_E \beta_R}{0.01}\right)^{1/7} \mu_{30}^{3/7} \text{ s}. \quad (17)$$

For  $P < P_{cr}$  the synchrotron energy loss rate is

$$\begin{aligned} \dot{E}_{syn} &= \int_{r_L}^{\infty} dr I_r E_r = \alpha_E B_L^2 r_L^2 c \\ &\approx 5.8 \times 10^{30} \left(\frac{\alpha_E}{0.1}\right) \frac{\mu_{30}^2}{P^4} \frac{\text{erg}}{\text{s}}, \end{aligned} \quad (18)$$

which is a fraction  $\alpha_E$  of the Goldreich-Julian power. For  $P < P_{cr}$  the kinetic energy flux of the particles is

$$\begin{aligned} \dot{E}_{kin} &= (\gamma_{max} - 1)mc^2 \frac{dN}{dt} \\ &\approx 7 \times 10^{31} \left(\frac{\alpha_E}{\beta_R}\right)^{1/2} \frac{\mu_{30}^{1/2}}{P^{1/2}} \frac{\text{erg}}{\text{s}}. \end{aligned} \quad (19)$$

For  $P < P_{cr}$  we have  $\dot{E}_{kin} / \dot{E}_{syn} \approx 12.2(P^{7/2} / \mu_{30}^{3/2}) < 1$  for  $\alpha_E = 0.1 = \beta_R$ .

For  $P > P_{cr}$  the synchrotron losses are small compared with power input to the particles in the neutral layer. The total synchrotron radiation from the neutral layer ( $r \geq r_L$ ) is

$$\begin{aligned} \dot{E}_{syn} &= \frac{2}{3} r_e^2 r_L \beta_R B_L^2 \gamma_0^2 \frac{dN}{dt} \\ &\approx 3.9 \times 10^{29} \beta_R \left(\frac{\alpha_E}{0.1}\right)^2 \frac{\mu_{30}^5}{P^{11}} \frac{\text{erg}}{\text{s}} \end{aligned} \quad (20)$$

The kinetic energy input to the particles in the neutral layer is  $\alpha_E B_L^2 r_L^2 c$ .

The synchrotron radiation of the particles accelerated in the neutral layer is beamed radially outward in the

equatorial plane of the pulsar. That is, the radiation is in a fan beam perpendicular to the pulsar's rotation axis. In reality, the neutral layer is probably not a thin layer with small width  $\Delta z$  everywhere. The accelerated particles will scatter from irregularities in the neutral layer and will interact with the magnetic field at larger distances,  $z \gg \Delta z$ . This process may explain the radiation from the disk-like pulsar wind nebulae such as those in the Crab and Vela nebulae.

For  $P < P_{cr}$  the frequency of the peak of the radiation spectrum is

$$\begin{aligned} \nu_{syn} &\approx \frac{1}{4\pi} \frac{e B_L \gamma^2}{mc} \left(\frac{r_L}{r}\right) \\ &\approx 1.3 \times 10^{22} \left(\frac{\alpha_E}{\beta_R}\right) \left(\frac{r_L}{r}\right) \text{ Hz}, \end{aligned} \quad (21)$$

where the lepton Lorentz factor is from equation (16). This frequency corresponds to a photon energy of  $\approx 54$  MeV. For  $P > P_{cr}$ , the Lorentz factor of the leptons is  $\gamma_0$  so that

$$\nu_{syn} \approx 8.5 \times 10^{19} \left(\frac{\alpha_E}{0.1}\right)^2 \frac{\mu_{30}^3}{P^7} \text{ Hz}. \quad (22)$$

This frequency corresponds to  $\approx 350$  keV. These numbers are of course estimates because the value of  $\alpha_E$  is not known.

## 5. THE MAGNETOTAIL

Beyond the light cylinder the pulsar relativistic wind carries the power  $\dot{E}_w = B_L^2 r_L^2 c$  in the electromagnetic field and relativistic particles (GJ69). The spin down of the pulsar is given by  $I\omega_* \dot{\omega}_* = -\dot{E}_w$  which implies that

$$P^2 = P_0^2 \left(1 + \frac{t}{\tau}\right), \quad \tau \equiv \frac{IP_0^2 c^3}{8\pi^2 \mu^2}, \quad (23)$$

where  $t$  is the age of the pulsar,  $P_0$  is the initial period of the pulsar, and  $\tau \approx 1.4 \times 10^{11} (P_0 / 0.02 \text{ s})^2 / \mu_{30}^2 \text{ s} \approx 4,300 (P_0 / 0.02 \text{ s})^2 / \mu_{30}^2 \text{ yr}$  assuming  $I = 10^{45} \text{ g cm}^2$ . After a time  $t$  the total energy put out by the pulsar is  $E_w = I\omega_{*0}^2 [t/(t+\tau)]/2$ , where  $\omega_{*0} = 2\pi/P_0$ .

The pulsar wind also carries toroidal magnetic flux outward. The rate of transport of positive toroidal flux is

$$\dot{\Phi}_+ = \pi B_L r_L c \approx 4.1 \times 10^{21} \frac{\mu_{30}}{P^2} \frac{\text{G cm}^2}{\text{s}}. \quad (24)$$

After a time  $t$  the total (positive) toroidal flux put out by the pulsar is  $\Phi_+ = (\pi \mu \omega_{*0}^2 \tau / c) \ln(1 + t/\tau) = (\pi/2) \mu I c^2 \ln(1 + t/\tau)$ . Of course the net flux is zero.

For early times after the pulsar's birth there is a spherical Sedov-Taylor "bubble" of plasma of radius  $\mathcal{R}(t) \propto t^a$  with  $a < 1$ . Here we consider *late times* in the sense that the pulsar has had time to move outside of this bubble. That is, we consider  $t > \mathcal{R}(t)/v$ , where  $v$  is the pulsar's velocity. This corresponds to  $t > 3 \times 10^9 \text{ s} (\mathcal{R}_{pc} / v_8)$ , where  $\mathcal{R}_{pc}$  is the bubble radius in pc, and  $v_8 \equiv v / 10^8 \text{ cm/s}$ .

Figure 3 shows a sketch of the late stage of the magnetotail of a neutron star moving supersonically with velocity  $v$  through the interstellar medium. For simplicity we have assumed that the star's velocity is parallel to its rotation axis  $\omega_*$ . The standoff distance of the shock  $z_{sh}$  is given by equation (3). The pulsar wind extends out

to a radial distance  $r_{m0} \approx z_{sh}$  from the  $z$ -axis at  $z = 0$ . The magnetic field strength at this distance is

$$B_{m0} = B_L \left( \frac{r_L}{r_{m0}} \right) = (4\pi\rho_{ism}v^2)^{1/2} \approx 4.6 \times 10^{-4} n_{ism}^{1/2} v_8 \text{ G}, \quad (25)$$

which is independent of the period and magnetic moment of the pulsar. The magnetic field pressure of the wind at  $r_{m0}$  is

$$p_{m0} = \frac{\mathbf{B}^2}{4\pi} \Big|_{r_{m0}} = \Gamma p_{ism} \mathcal{M}^2, \quad (26)$$

where  $p_{ism}$  is the ambient pressure of the interstellar medium,  $\Gamma$  is the ratio of specific heats, and  $\mathcal{M} = v/c_s$  is the Mach number of the pulsar. Typically,  $\mathcal{M} \gg 1$ , so that electromagnetic pressure at  $r_w$  is much larger than  $p_{ism}$ . The pressure of the shocked interstellar gas is larger than  $p_{ism}$  by a factor  $\mathcal{M}^2$  neglecting radiative cooling. Because the time scale  $t_m \equiv r_{m0}/v$  is much shorter than the spin-down time scale  $\tau$ , the time-dependence of the pulsar's period can be neglected.

The momentum of the relativistic pulsar wind is *deflected* by the *internal shock* as indicated in Figure 3. The flow of power and magnetic flux is however essentially unchanged. We assume that the internal shock has the effect of establishing an *approximate equipartition* between the field energy density and the energy density of relativistic particles. It is unlikely that the pulsar's magnetized wind is compressed into a thin layer in contact with the interstellar medium as discussed in the hydrodynamic model (Baranov, Krasnobaev, & Kulikovskii 1971; Wilkin 1996). Plasma in the magnetotail moves away from the pulsar with an initially *relativistic* velocity,  $\mathbf{v}_m \approx v_{mz} \hat{\mathbf{z}}$ , where  $v_{mz}(z=0) = \mathcal{O}(c)$ . As indicated in Figure 3, the magnetic field in the magnetotail is predominantly toroidal,  $\mathbf{B} \approx \hat{\phi} B_m(z)$ .

Inside the magnetotail there is a cylindrical neutral layer across which the magnetic field reverses direction (see also simulations by Romanova et al. 2001; Toropina et al. 2001). Reconnection or annihilation of the magnetic field in this layer continually accelerates the charged particles of the flow. In the following equations, we assume that the charged particles consist of electrons and positrons. The flux of particles from the pulsar is  $dN_\ell/dt \approx 2B_L r_L c/e$ . In steady state this particle flux flows down the magnetotail so that  $dN_\ell/dt = \pi r_m^2 n_\ell v_{mz}$ , where  $n_\ell$  is the lepton density. The equipartition Lorentz factor of the leptons is

$$\begin{aligned} \langle \gamma_\ell \rangle &= \frac{1}{8} \frac{e B_L r_L}{m c^2} \left( \frac{B_m r_m}{B_{m0} r_{m0}} \right)^2 \frac{v_{mz}}{c}, \\ &\approx 10^6 \frac{\mu_{30}}{P^2} \left( \frac{B_m r_m}{B_{m0} r_{m0}} \right)^2 \left( \frac{v_{mz}}{10^{10} \text{ cm/s}} \right), \end{aligned} \quad (27)$$

where we have used the fact that  $B_{m0} r_{m0} = B_L r_L$ .

The total, kinetic plus field, energy flux along the magnetotail is

$$\mathcal{F}_E(z) = \frac{1}{2} (B_m r_m)^2 v_{mz}. \quad (28)$$

Assuming a *stationary flow* with respect to the pulsar, the synchrotron losses along the magnetotail imply

$$\frac{d\mathcal{F}_E}{dz'} = -(\pi r_m^2 n_\ell) \frac{2}{3} r_e^2 c B_m^2 < \gamma_\ell^2 >, \quad (29)$$

or

$$\frac{d(\chi^2 v_{mz})}{dz'} = -\frac{1}{z_0} \left( \frac{r_{m0}}{r_m} \right)^2 \chi^6 v_{mz}, \quad (29)$$

where  $z' \equiv -z \geq 0$ , and

$$\chi \equiv (B_m/B_{m0})(r_m/r_{m0}) \leq 1.$$

Here,  $z_0$  is the *radiation length-scale* of the magnetotail,

$$\begin{aligned} z_0 &= \frac{24m^2 c^4}{B_L r_L r_e^2 e (4\pi\rho_{ism}v^2) \mathcal{C}_\ell}, \\ &\approx 15 \frac{P^2}{\mu_{30} n_{ism} v_8^2 \mathcal{C}_\ell} \text{ pc}, \end{aligned} \quad (30)$$

and  $\mathcal{C}_\ell \equiv \langle \gamma_\ell^2 \rangle / \langle \gamma_\ell \rangle^2$  is a constant of order unity. Solutions of equation (29) for  $v_{mz} = \text{const}$ , indicate that  $\chi$  decreases very gradually with  $z'$ .

The frequency of the peak of the synchrotron radiation from the magnetotail is

$$\begin{aligned} \nu_{syn} &\approx \frac{1}{4\pi} \frac{e B_m \gamma^2}{mc} \\ &\approx 7.4 \times 10^{14} \frac{\mu_{30}^2 n_{ism}^{1/2} v_8}{P^4} \chi^5 \left( \frac{r_{m0}}{r_m} \right) \left( \frac{v_{mz}}{10^{10} \text{ cm/s}} \right)^2 \text{ Hz}, \end{aligned} \quad (31)$$

where we have used equation (27) for lepton Lorentz factor.

Most pulsar velocities through the ISM are less than  $10^3$  km/s, with typical velocities  $\sim 100 - 500$  km/s (Arzoumanian, Chernoff, & Cordes 2002). For these slower pulsars, the length  $z_0$  may be larger than the total distance the pulsar has travelled. For such conditions only a small fraction of the magnetotail energy flux  $\mathcal{F}_E$  is lost to synchrotron radiation. Here we discuss the long distance  $z' \gg r_{m0}$  behavior of the magnetotail in this limit. The internal pressure of the magnetotail,  $p_m$ , is much larger than the pressure of the ISM at the distance  $z'$ ,  $p_{ism} = \rho_{ism} c_s^2 / \Gamma \ll p_m$ . In general the radius of the magnetotail increases with distance from the star. Assuming a stationary configuration in the reference frame moving with the star we have

$$\begin{aligned} \frac{dr_m(z')}{dz'} &= \frac{v_{mr}}{v_{mz}}, \\ v_{mr} &= \left( \frac{p_m(z')}{\rho_{ism}} \right)^{1/2} = \left( \frac{\dot{E}_w}{3\pi r_m^2 v_{mz} \rho_{ism}} \right)^{1/2}. \end{aligned} \quad (32)$$

Thus

$$\begin{aligned} \frac{dr_m(z')}{dz'} &= \left( \frac{4c}{3v_{mz}} \right)^{1/2} \frac{r_{m0}}{r_m} \frac{v}{v_{mz}}, \\ &\approx 0.02 v_8 \left( \frac{r_{m0}}{r_m} \right) \left( \frac{10^{10} \text{ cm/s}}{v_{mz}} \right)^{3/2}. \end{aligned} \quad (33)$$

Expressed in this form,  $dr_m/dz'$  is independent of  $\dot{E}_w$  and  $\rho_{ism}$ . Equation (32) represents a balance of the internal pressure of the magnetotail,  $p_m(z')$ , against the ram pressure due to the tail's expansion into the ISM,  $\rho_{ism}(dr_m/dt)^2$ , where  $dz'/dt = v_{mz}$ .



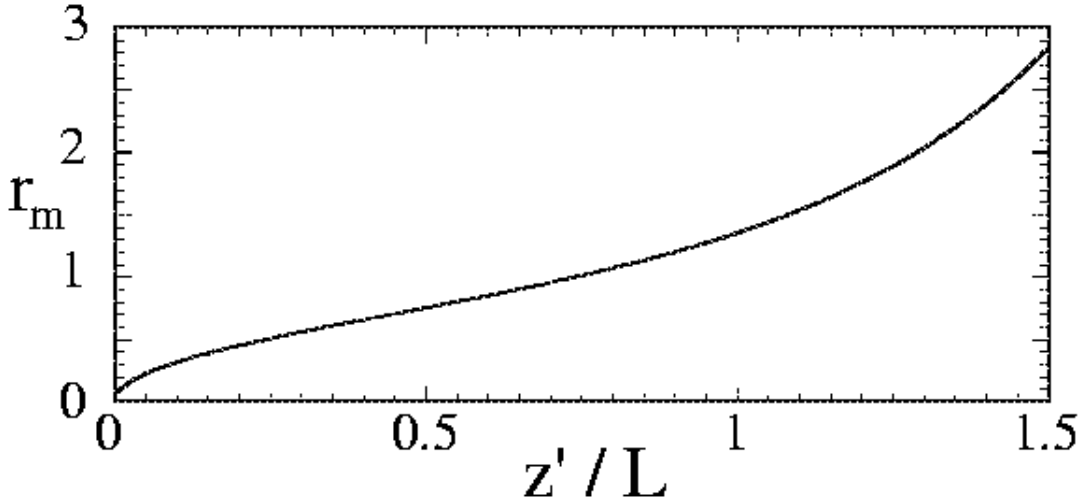


FIG. 4.— Qualitative dependence of the width of the magnetotail  $r_m(z')$  on the distance behind the star  $z'$ . The vertical scale is arbitrary; it given by  $(z'/L)^{1/2}F(z'/L)$ , with  $F$  given below equation (34).

For distances  $z' \gg r_{m0}$  but small compared with the total distance the star has travelled,  $v_{mz}$  is approximately constant. Consequently,

$$r_m(z') \approx 0.2 (r_{m0} z')^{1/2} \left( \frac{10^{10} \text{cm/s}}{v_{mz}} \right)^{3/4}, \quad (34)$$

for  $z' \gg r_{m0}$ .

The situation is different for distances  $z'$  of the order of or somewhat larger than the total distance travelled by the star,  $L \equiv vt \approx 10v_8(t/10^4 \text{yr}) \text{pc}$ , with  $t$  the age of the star. The velocity of the magnetotail plasma  $v_{mz}$  must decrease strongly with  $z'$  of the order of this distance owing to collisions with the plasma of the above-mentioned bubble  $\mathcal{R}(t)$  formed at the birth of the pulsar. For a rough description consider

$$v_m(z') = v_m(0) \exp \left( -\frac{(z')^2}{L^2} \right).$$

For this case equation (33) gives

$$r_m(z') = 0.2 (r_{m0} z')^{1/2} \left( \frac{10^{10} \text{cm/s}}{v_{mz}(0)} \right)^{3/4} F \left( \frac{z'}{L} \right), \quad (35)$$

where

$$F(\xi) \equiv \left( \frac{1}{\xi} \int_0^\xi d\xi \exp(3\xi^2/2) \right)^{1/2}.$$

Figure 4 shows the qualitative dependence of  $r_m(z')$ . Notice that the curvature of the magnetotail radius  $d^2 r_m / dz'^2$  is initially negative but at large  $z' \gtrsim 0.45L$  it becomes positive. Further note that  $v_{mz}(z')[r_m(z')]^2$  is an increasing function so that  $B_m(z')$  decreases with distance from the star.

The magnetotails may be observable as low surface brightness regions of non-thermal, polarized (synchrotron) emission.

## 6. EXAMPLES OF MAGNETOTAILS: GUITAR AND MOUSE NEBULAE

The Guitar Nebula is created by the high velocity pulsar B2224+65. Chatterjee and Cordes (2002, 2004) estimate the velocity of the pulsar as  $v_8 \approx 1.7$  or 1700 km/s based on a distance to the source of  $D = 1.9$  kpc. The pulsar period  $P = 0.68$  s and spin-down rate  $\dot{P} = 9.7 \times 10^{-15} \text{s/s}$  imply a power output  $\dot{E} \approx 1.2 \times 10^{33} \text{erg/s}$  assuming a moment of inertia  $I = 10^{45} \text{g cm}^2$ . The angular length of the tail is  $\delta\theta \approx 15''$  (Chatterjee & Cordes 2004) which corresponds to a tail length  $z' \approx 0.15 \text{pc}/\sin \iota$ , where  $\iota$  is the angle between the jet axis and the line of sight. With  $\dot{E} = B_L^2 r_L^2 c$ , this power implies  $\mu_{30} \approx 2.1$ . Consequently, equation (30) gives the length-scale of the magnetotail  $z_0 \approx 1 \text{pc}/(n_{ism} \mathcal{C}_\ell)$ . This corresponds approximately to the whole length of the tail in the Guitar nebula observed in the  $H_\alpha$  line. Equation (31) gives the estimate  $\nu_{syn} \approx 2.6 \times 10^{16} \text{Hz}$   $n_{ism}^{1/2} \chi^5 (r_{m0}/r_m) (v_{mz}/10^{10} \text{cm/s})^2$ . The Guitar nebula was observed as a source of very weak radiation in soft X-rays which is a sign of the presence of highly relativistic leptons,  $\gamma \sim 10^7$ . (Romani, Cordes, & Yadigaroglu 1997). Equation (13) also gives similar  $\gamma$  for the Guitar nebula. Both, the magnetic field and the particles interact with the interstellar medium and are deflected to the tail. The radio images of the Guitar nebula show clear limb-brightening (e.g., Chatterjee & Cordes 2004). This may be due to the compression of the magnetic field at the shock wave which leads to higher intensity of radiation (e.g., Romani et al. 1997). From other side, enhanced reconnection is expected near the shock wave due to compression the magnetic field components of opposite polarity. This will also lead to limb-brightening. The inner (smaller scale) structure of the Guitar nebula is closed in the back and may represent the inner shock wave, where accelerated particles

were stopped by the interstellar medium. If this is the case, then the flow has  $\sigma < 1$  before the shock wave.

Another example is a Mouse Nebula which is created by much more powerful pulsar PSR J1747-2958 which propagates with high velocity  $v \approx 600$  km/s (Camilo et al. 2002). It has a period  $P = 0.098$  s and a spin-down luminosity  $\dot{E} \approx 2.5 \times 10^{36}$  erg/s which corresponds to  $\mu_{30} \approx 2.5$ . The extended radio nebulae was discovered around this pulsar (Yusef-Zadeh & Bally 1987). An interesting feature is the tail which propagates to very far distances. Recently, an X-ray nebula was found in observations with Chandra (Gaensler et al. 2004). Gaensler et al. (2004) estimate the X-ray luminosity of the nebula as  $L_X(0.5 - 8 \text{ keV}) \approx 5 \times 10^{34}$  erg/s based on a distance to the source of  $D = 5$  kpc, the angular length of the Mouse tail as  $\delta\theta \approx 45''$  which corresponds to a tail length  $z' \approx 1 \text{ pc}/\sin\iota$ . For these parameters and assuming  $v_8 = 0.6$  and  $n_{ism} = 0.3/\text{cm}^3$  (Gaensler et al. 2004), equation (30) gives radiation length-scale of the magnetotail,  $z_0 \approx 0.5 \text{ pc}/C_\ell$ . For this object equation (31) gives the estimate  $\nu_{syn} \approx 2.7 \times 10^{19} \text{ Hz}$   $n_{ism}^{1/2} \chi^5(r_{m0}/r_m)(v_{mz}/10^{10}\text{cm/s})^2$ . The predicted frequency and overall luminosity is much higher than those for Guitar nebula, which correspond to observations of much brighter X-ray source.

The shape of the Mouse nebula was modelled by hydrodynamic simulations (Gaensler et al. 2004; van der Swaluw et al. 2003; and Bucciantini 2002). It was suggested that the inner brightest region of the nebula may be due to the termination shock of the pulsar wind, while the long tail may represent the post-shock flow. We note that the Chandra observations do now show evidence of the shock wave in the X-ray picture of this nebula. Instead, the X-ray luminosity in the brightest region (the head of the Mouse) varies gradually (Gaensler et al. 2004). This may be a sign that in the tail the  $\sigma$  parameters is not small and the back flow is not super-Alfvénic. If  $\sigma \sim 1$  in the tail, then the magnetotail may propagate to very far distances (see, e.g., simulations by Romanova et al. (2001) and Toropina et al. (2001). Radio radiation is then connected with particles which propagate from the pulsar along the tail to far distances and lose their energy there, or, particles may be accelerated in situ during the reconnection processes in the magnetotail (Toropina et al. 2001).

The direction of jets observed in a few cases is almost aligned with the direction of propagation of the pulsar and seems to be perpendicular to the PWN disk (e.g., Ng & Romani 1987). If the direction of the pulsar's motion is determined by a magnetic kick during its formation (Ardeljan, Bisnovatyi-Kogan & Moiseenko 2001; Lai, Chernoff & Cordes 2000) then both,  $\omega_*$  and  $\mu$  will be in the same direction. If the magnetic axis is misaligned relative to the rotational axis, but  $\omega_*$  is aligned with the pulsar's velocity  $\mathbf{v}$ , then the situation is similar to the one described above, because the jet forms in the direction of  $\omega_*$  and the disk magnetic structure is perpendicular to  $\omega_*$ . However, if  $\omega_*$  does not coincide with  $\mathbf{v}$ , then both the jet and the PWN disk may give non-axisymmetric features of the pulsar wind nebula. This later situation has not been investigated. A related situation appears in the recently discovered double pulsar binary PSR J0737-3039 where the direction of the flow

from the wind-generating pulsar changes in time because of the motion of stars in the binary system (Kaspi et al. 2004; Demorest et al. 2004). Three dimensional simulations of such a flow have shown that for large inclination angles between magnetic and rotational axes of pulsar, the shock wave may be non-axisymmetric (Spitkovsky & Arons 2004).

## 7. OBSERVABILITY OF WIND AND MAGNETOTAIL EMITTING STARS IN THE SOLAR NEIGHBORHOOD

Although neutron stars which spin-down to periods  $P \gtrsim 1 - 3$  s shut off as pulsars, they may continue to emit a wind and interact with the ISM. The number of such objects is larger than the number of short period pulsars. It is possible that such objects are detectable at distances  $\lesssim 10^2$  pc. Interesting nearby isolated neutron star candidate RX J1856.5-3754 has a prominent shock wave, and we discuss this object below.

Different spin-down powers  $\dot{E}_w$  are expected depending on the star's period of rotation and magnetic field. We consider two cases, one with a relatively high spin-down power  $\dot{E}_w \sim 10^{31} - 10^{32}$  ergs/s which is pertinent to old pulsars, or relatively powerful shutoff pulsars, and a low spin-down power,  $\dot{E}_w = 10^{28}$  ergs/s corresponding to very weak shutoff pulsars.

### 7.1. Relatively High Spin-Down Power Objects

Shutoff or misdirected pulsars with periods  $P \sim 1$  s and a surface magnetic field  $B \sim 10^{12}$  G have a relatively high spin-down power,  $\dot{E}_w \sim 6 \times 10^{31}$  erg/s (i.e., equation 2). The synchrotron emission from the pulsar wind for distances within the bow shock wave is given by equation (20) and the photon energies are given by equation (22) and are in the high X-ray range. Additionally, there will be a small power in thermal radiation from the shocked ISM according to equation (5). For high velocity pulsars, say  $v > 500$  km/s, a significant fraction of the spin-down power is emitted in the magnetotail. The photon energies are then much lower, in the UV band according to equation (31). Such shut-off or misdirected pulsars may be observed owing to the radiation of their tails.

### 7.2. Nature of RX J1856.5-3754

The source RX J1856.5-3754 is an interesting isolated neutron star candidate located at a distance  $d \approx 60$  pc (Walter 2001). It has a bow shock wave observed by van Kerkwijk and Kulkarni (2001a,b) in the  $H_\alpha$  line. The observed standoff distance between the star and the bow shock wave is  $\approx 1''$  which corresponds to  $z_{sh} \approx 0.9 \times 10^{15}$  cm. The bow shock is probably due to the interaction of star's wind with the ISM (see, e.g., van Kerkwijk and Kulkarni 2001b). In this case we obtain from equation (3) the relation  $B_{12}/(n_{ism}^{1/2} v_8 P^2) \approx 9$ , where  $B_{12}$  may be replaced by  $\mu_{30}$ . This relation implies that the spin-down power of the star is  $\dot{E}_w \approx 5.2 \times 10^{32} n_{ism} v_8^2$  erg/s  $\approx 5.2 \times 10^{30}$  erg/s, where we took  $v_8 = 0.1$  and  $n_{ism} = 1/\text{cm}^3$ .

Clearly, a range of values of  $B_{12}$  (or  $\mu_{30}$ ),  $P$ , and  $v_8$  can give  $\mu_{30}/(v_8 P^2) \approx 9$ , where we have taken for simplicity  $n_{ism} = 1$  ( $1/\text{cm}^3$ ). To consider the likely values, note that the spin-down time of the star is  $t_{sd} \equiv P/\dot{P} = IP^2 c^3/(4\pi^2 \mu^2) \approx 2.2 \times 10^7 \text{ yr } P^2/\mu_{30}^2$ ,

assuming  $I = 10^{45} \text{ g cm}^2$ . The total number of neutron stars likely to be within a distance of 60 pc of the Sun is  $N \approx 10^9 (60 \text{ pc} / 10^4 \text{ pc})^3 \approx 220$  (e.g., Schvartsman 1978; Treves & Colpi 1991). Then the probable number of objects with spin-down time  $t_{sd}$  within 60 pc is  $N' \approx 220(t_{sd}/t_H)$ , where  $t_H \approx 1.5 \times 10^{10}$  years is the Hubble time. Thus, for the object RX J1856.5-3754,  $N' \approx 0.036/(\mu_{30}v_8)$ . The corresponding rotation period of the star is  $P \approx 0.33(\mu_{30}/v_8)^{1/2}$  s. Clearly, larger values of  $N'$  occur for a smaller values of the product  $\mu_{30}v_8$ . For example, for  $\mu_{30} = 0.1$  and  $v_8 = 0.1$ ,  $N' \approx 3.6$ . For these values  $\dot{E}_w \approx 5.2 \times 10^{30} \text{ erg/s}$ . So, this object may be a misdirected pulsar if magnetic field is large, or, shutoff pulsar if the field is small.

### 7.3. Low Spin-Down Power Objects

There should be a much larger number of shutoff pulsars with the lower spin-down power. As an example, we take objects with the spin-down power  $\dot{E}_w = 10^{28} \text{ erg/s}$ . For these objects,  $\mu_{30}^2/P^4 \approx 1.7 \times 10^{-4}$ , so that  $P \approx 8.7\mu_{30}^{1/2}$  s. The number of such objects within a distance of 60 pc of the Sun is  $N' \approx 220(t_{sd}/t_H) \approx 24/\mu_{30}$ . The standoff distance of the bow shock wave is however small,  $z_{sh} \approx 1.3 \times 10^{12} v_8^{-1} \text{ cm}$ , but much larger than the light cylinder radius.

## 8. DISCUSSIONS AND CONCLUSIONS

This work considers rotating magnetized neutron stars emitting power  $\dot{E}_w$  in a wind of relativistic particles and electromagnetic field. We argue that a non-negligible fraction of the wind's power may be converted to relativistic particles due to annihilation or reconnection of the magnetic field. Outside of the light cylinder, the star's rotation acts to wind up the magnetic field to form a predominantly azimuthal, slowly decreasing with distance, magnetic field of opposite polarity on either side of the equatorial plane normal to the star's rotation axis. An analogous situation exists in the Solar Wind (Bertin & Coppi 1985). The magnetic field annihilates across the equatorial plane with the magnetic energy going to accelerate the charged particles to highly relativistic energies. The accelerated leptons emit synchrotron radiation in a broad range from the UV to gamma ray energies. Additionally, there may be oppositely directed Poynting outflows along the star's rotation axis. These outflows

are analogous to the Poynting jets discussed by Lovelace and Romanova (2003). The model is qualitative in the respect that we do not have global self-consistent calculations of the electromagnetic fields and sources.

For a typical, supersonically moving star, the star's relativistic wind forms a bow shock wave with the interstellar medium (Schvartsman 1970). We argue that an appreciable fraction of the star's wind power  $\dot{E}_w$  is deflected by the bow shock wave and collimated into the star's magnetotail. Plasma moves down the magnetotail with a relativistic velocity. Further, we argue that equipartition is reached in the magnetotail between the magnetic energy and the relativistic particle energy. For the case where the charged particles are leptons, the synchrotron radiation length-scale  $z_0$  of the magnetotail is calculated. Over this distance the energy flux in the magnetotail decreases by a factor of order 2. For highly supersonic pulsars,  $z_0$  may be less than the total distance the star has travelled. The synchrotron radiation spectrum is expected to be nonthermal with typical photon energies in the UV range for the highly supersonic pulsar B2224+65 which generates a bow shock wave and the Guitar Nebula (Chatterjee & Cordes 2002).

The ratio of the power due to the shock heating of the ISM, which gives thermal emission, to the wind power  $\dot{E}_w$ , which gives nonthermal synchrotron emission, is shown to be  $(3/64)(v/c) \ll 1$ , where  $v$  is the velocity of the star and  $c$  the speed of light.

An equation is derived for the radius of the magnetotail  $r_m(z')$  as a function of distance  $z'$  from the star. For large distances  $z'$ , of the order of the distance travelled by the star, we argue that the magnetotail has a 'trumpet' shape owing to the slowing down of the magnetotail flow.

We argue that the isolated neutron star candidate RX J1956.5-3754 may be a misdirected or shutoff pulsar.

We estimate the number of shutoff pulsars which may be observable in the vicinity of the Sun for cases of relatively strong ( $10^{32} \text{ erg/s}$ ) and weak ( $10^{28} \text{ erg/s}$ ) power. A much larger number of weak shutoff pulsars is predicted.

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