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Abstract. Since the Stanford pioneering work of Paik in the 1970s, cryogenic resonant-mass gravitational wave detectors have used resonant transducers, which have the effect of increasing both the detector sensitivity and bandwidth. Now nanotechnology is opening new possibilities towards the construction of ultra-high sensitivity klystron cavity transducers. It might be feasible to construct TeraHz/micron parametric transducers in a near future. They would be so sensitive that there would be no need for multimode resonant transducers. The resonant-antenna would act as a broadband detector for gravitational waves. A spherical antenna, such as Schenberg or Mini-Grail, could add to this quality the advantage of wave position and polarity determination. Here we propose an extreme geometry for a re-entrant klystron cavity (df/dg ~ 10¹⁸ Hz/m, where f stands for the microwave pump frequency and g for variations in the cavity gap), obtaining a frequency response for the strain sensitivity of the Schenberg gravitational wave detector such that its bandwidth increases from 50 Hz (using the so-called resonant mode coupling) to ~4000 Hz when operating @ 20 mK, and, when compared to LIGO experimental curve, shows a competitive band of about 2000 Hz. We also study some of the technological complications that can be foreseen to design such a resonant cavity.

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1. Introduction

Pioneered by Weber in the 1960s, resonant-mass gravitational wave detectors were the first devices developed for the purpose of detecting propagative perturbations of the space-time metric. Since then, there have been many similar devices developed throughout the world, with the geometry of bars and spheres. This work is part of the Brazilian group’s resonant-mass detector program to improve the sensibility of the Schenberg detector, which had its first data run in September 2006 [1].

All resonant-mass detectors in operation [1] have narrow bandwidths when compared to interferometric detectors, such as LIGO [2]. This is due to large electronic noise of the transductance mechanism that converts the motion of the detector to an electronic signal. In order to enhance the
bandwidth, it was proposed by Paik [4] that the transducer could be constructed from coupled
harmonic oscillators, with masses in such a progression that there would be high amplitude gain of the
antenna motion with the frequency response naturally leading to a higher bandwidth by raising the
signal above the electronic transducer noise. However, the trade-off is that this technique creates an
intrinsic bandwidth due to the mechanical thermal noise added by the transducer modes, which cannot
be surpassed and is still much lower than the typical interferometer bandwidth.

We will propose here a novel approach to the problem. New nanotechnologies have been enhanced
during the past years and can now produce extremely smooth surfaces; such that just a few nanometers
of roughness can be produced. In such a scenario, we propose a nanometric-gap reentrant klystron
cavity, such that, when operated at extremely low temperature (@20 mK) will give such low
electronic noise that the resonant-mass transducer may be dispensed with, and a bandwidth
competitive to interferometer gravitational wave detectors may be obtained.

In section two, we give an overview of the klystron cavity, the noise analysis which is responsible
for the strain sensitivity of the detector, and the design parameters that give the expected behavior, and
compare our obtained curve with the one obtained from LIGO I fourth run of data [2]. In section 3 we
outline a method to build this nano-cavity, together with possible problems in its operation and
fabrication.

2. The Klystron Cavity Transducer and Noise Analysis
The klystron cavity has a reentrant geometry which is depicted in Figure 1. Its characteristics are well-
known for electronics engineers since Hansen’s work [5]. The gap, whose dimension is $g$, determines
the dominant mode, i.e., the well-known klystron mode, which makes the gap resemble a capacitor,
with the electric field lines running across the gap spacing between the top of the conical post to the
top of the cavity and vice-versa, with the magnetic field lines circulating the region around the conical
post (this is the inductive region of the cavity).

![Figure 1. The klystron cavity transducer geometry and its relevant dimensions. The cavity that we devise is depicted to scale in a 3D perspective.](image)

The sensitivity of the detector is due to four main types of noise (we neglect the effect of seismic
noise, as our lowest frequency of interest is about 2 kHz); these are the transducer series noise, the
oscillator phase noise, the back-action noise and mechanical thermal noise [6]. Some electrical
parameters that are directly related to the noise sources are the electrical Q-factor, and the cavity
dominant frequency dependence on the gap dimension, $df/dg$. When using superconducting niobium
and a gap of tenths of µm, we have usual values of: $df/dg \approx f_0/g \approx 10^{15}$ and $Q_e \approx 10^5$ [1] ($f_0$ is the klystron
mode frequency assumed to be 10 GHz).
Our proposal is to adjust these two factors to extremely high values, so that the noise produced by the klystron cavity transducer is sufficiently low that mechanical amplification of the gravitational wave signal via a multi-mode transducer is not necessary. This will eliminate the necessity of having the top of the cavity made of an oscillating membrane tuned to the sphere frequency. Instead we need to design an inertial mass.

To design this cavity, we used the work from Fujisawa [7]. For small gaps (which is, indeed, our case) he derived some semi-analytical expressions for cavity frequency as function of its geometry, using a lumped circuit analogous to the cavity. Using a Mathematica® notebook, we could design the transducer to maintain its dominant mode at 10 GHz and have the desired \( \frac{df}{dg} \) of about \( 10^{18} \) Hz/m. The cavity dimensions found are listed below.

\[
R = 8 \ \text{mm}; \\
r_{\text{ext}} = 27 \ \text{\mu m}; \\
r_{\text{ext}} = 500 \ \text{\mu m}; \\
h = 2 \ \text{mm}; \\
g = 1 \ \text{nm};
\]

Assuming these parameters and operation at 20 mK, we investigate what will happen to the strain noise curve of the Schenberg detector when subjected to these conditions.

The behavior of the detector under these transducer conditions is depicted in Figure 2. It shows the known noise components of the Schenberg’s sensitivity curve, and why it becomes broadband: the transducer phase and series noise components are greatly decreased by the increase of \( \frac{df}{dg} \), so that the resonant peak becomes much less relevant to the final curve, which has just a little concavity in the resonance region.

![Figure 2. The Schenberg detector sensitivity curve is depicted in a black, thick, curvy line (the flat line just below it represents the thermal noise and the other flat line the back-action noise). The gray line with a dip represents series and phase noises when viewed at the sphere input (the same port where the gravitational wave arrives). The blue line is an interpolation from LIGO October 2006 sensitivity curve [2].](image)
From Figure 2, we can see in the band from 2500 Hz to 4500 Hz Schenberg would have a better sensitivity than LIGO, and in the band from 2000 Hz to 6000 Hz Schenberg would be comparable and only lose in sensitivity by a factor of at most 2.

The observation of sources such as stellar black holes and neutron stars would benefit from this sensitivity enhancement of spherical antennas in comparison with interferometric techniques.

There is, however, a great technological challenge in order to obtain such a cavity. In the following section we outline our proposed method to obtain the nanometre gap, and list some of the difficulties that will eventually arise during the process of its fabrication.

3. Building a nanometric Klystron Cavity Transducer

Nanotechnology deals with tiny distances, on the nanometre scale. But what we wish to do is to give birth to an extremely little gap even for nanotechnology procedures. Our proposed technique is as follows.

The bulk of the cavity is composed of silicon, in whose surface is deposited a niobium layer, which is superconductor at the extremely low temperatures of Schenberg operation (today, Schenberg operates at 4K, being cooled by liquid helium). There is thermal contraction of those materials during cooling. The thermal contraction of niobium is about ten times larger than the one for silicon and as result the differential contraction coefficient of niobium related to silicon from room temperature to 4 K is about one part in a thousand, which means that, if there is a niobium layer 0.5 micrometer thick on the top of the cavity post, which, on the other hand, is touching another 0.5 micrometer thick layer deposited on the membrane above, at 300 K, when cooled down to 4 K, both niobium layers will suffer a differential contraction of 1/1000, making our desired gap, of 1 nanometre. This situation is depicted in Figure 3.

![Diagram of the klystron cavity](image)

Figure 3: The klystron cavity is made of two pieces of silicon, which are green in the figure. Both of them have a niobium layer of 0.5 micrometer (in red), and then they are put in contact, and sealed by silicon plates (in blue). When cooled down to ~4K, the niobium layer contracts one part in a thousand while the silicon one only contracts a part in ten thousand, therefore a gap of about one nanometre opens between the two niobium layers. In the picture at the right side, we show schematically those layers, at an electrically irrelevant point, that is, the point of contact between the cavity and the silicon plate.

There are several problems related to the construction of the cavity, and we list some of them below, proposing some ways to solve each problem.
3.1. Casimir Force
The Casimir force between the plates can be a serious problem because it can prevent the gap from appearing during the cooling of the cavity. Calculating Casimir forces analytically can be quite a problem if the geometry is irregular like ours, and if you don’t know a general expression for the cavity eigenmodes. The cavity vacuum state in Fock space becomes dependent of numerical parameters, and the ordinary way to calculate the vacuum energy is by using some regularization procedure, which involves the knowledge of the analytical expression for all the modes that can populate the cavity.

We will make a detailed analysis of the consequences of the Casimir force in a future paper.

3.2. Surface Roughness
The surface of the niobium must be very well polished, to a precision of a few tenths of nanometers, which is ten times better than we can achieve with present technology. So, there is a technological need to improve those methods by making surfaces roughness become lesser than half a nanometre, which is a big challenge because we begin to deal with the atomic scale.

3.3. Island Weight
The island is placed on the middle of the membrane to make it oscillate at much lower frequency than the quadrupole modes of the sphere. In order to make this membrane as an inertial reference body, we construct the island in such a way that the membrane oscillates at a frequency about ten times lower than the resonant frequencies of the quadrupole modes of the sphere, that is, ~ 320 Hz.

If we compare the restoring force from the membrane plus island to its weight, knowing that it can bend no more than tenths of nanometres, we come to the conclusion that the only way to maintain the gap opened without the interference of the membrane plus island weight is by keeping it in a vertical position.

During operation of the detector, it would be necessary to have a closed loop control of the vertical position of the cavity, where our actuator would be a piezoelectric device controlled by a noiseless DC voltage.

3.4. Electric Field Breakdown
This will not be a problem, as we can maintain the relation of the applied voltage to the gap of the cavity a constant throughout the operation, maintaining it in a value much lower than the threshold value to break the vacuum dielectric. Fortunately the electrical mechanical coupling is independent of the gap as long as voltage/gap ratio is maintained.

4. Conclusion
In this paper, we proposed a method to give resonant-mass detectors the ability to probe the gravitational universe in a wider range of frequencies. If this investigation really turns into a practical result, gravitational wave observation can become most easily accessible to groups throughout the world, as the costs are much lower for the resonant-mass techniques.

There remain many technological obstacles, and the existence of an attractive Casimir Force can be a potential problem to construct the nanometre gap by differential thermal contraction. In such a case, a new technique must be developed, maybe using the weight of the island to make it bend before thermal contraction begins.

Further work in a deeper analysis of the technical problems of the process of fabrication of the nanometre cavity, as well as a most realistic strain noise sensitivity curve, including the seismic effects and the other modes of vibration of the sphere remain to be done.
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References
[1] Aguiar et al 2008 Class Quantum Grav. 25 to be published
Aguiar et al 2006 Class Quantum Grav. 23 S239-S244
[2] Fafone, V 2004 Class Quantum Grav. 21 S377
[7] Fujisawa, K 1958 IRE Trans. Microwave Theory Tech. 6 344