The dark energy–dominated Universe

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Abstract

In this paper we investigate the epochs in which the Universe started accelerating and when it began to become dark energy–dominated (i.e., the dynamics of the expansion of the Universe dominated by the dark energy). We provide analytic expressions to calculate the redshifts of these epochs as a function of density parameters. Moreover, we review and discuss cosmological models with a dark energy component, which can have an interesting characteristic, namely, they never stop accelerating. This holds even if the Universe is at present time either flat, open, or closed. If the dark energy is the cosmological constant the Universe will eventually end up undergoing an exponentially expansion phase, and the total density parameter converging to $\Omega = 1$. This is exactly what is considered in inflationary scenario to generate the initial conditions for the big bang. One can then argue that the Universe begun with an inflationary phase and will end up with another inflationary phase. Thus, it follows that in both the early and the late Universe $\Omega \to 1$. We also discuss the above issues in the context of the XCDM parametrization.
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1. Introduction

There are many pieces of evidence for the existence of a component of the Universe other than the baryonic matter and the non baryonic dark matter (see, e.g., [5]).

This other component of the Universe could be Einstein's cosmological constant, or a component that varies slowly with time and space. Many authors name this component as dark energy or quintessence, should it be a cosmological constant or a component that acts like it.

From the measurements of the cosmic microwave anisotropy, the WMAP satellite team has recently announced that there is convincing evidence that most of the energy in the Universe today is dark. This dark energy is gravitationally repulsive

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and accelerates the expansion of the Universe (see, e.g., [8]), and could well be Einstein's cosmological constant.

It is worth mentioning that earlier evidence for an accelerating Universe was found by other astrophysical projects; The Supernova Cosmology Project is an example (see, e.g., [6]).

Also, it is worth recalling that the existence of a dark energy component is required for the age of the Universe be compatible with the oldest objects of the Universe (see, e.g., [1], amongst others, for a detailed discussion).

Although implicity in previous papers (see, e.g., [9,7,1,3,2], among others), and also in some textbooks (see, e.g., [4]), there is still some room to discuss some interesting questions not yet addressed to the same extent in the literature. Firstly, the epoch when the Universe begins accelerating. Secondly, the epoch when the Universe begins to be dark energy dominated. Thirdly, even Universe models spatially closed can expand forever. Fourthly, besides expanding forever spatially open, and closed models behave like flat models for a sufficiently long time; this very fact leads to an interesting conclusion, namely: the Universe begins almost flat and after being open, flat, or closed at present time, it will be flat in the future.

Since the dynamics is dominated by the dark energy, in the case of a cosmological constant, the scale factor increases exponentially for the late Universe. An interesting fact in such a case is that the scale factor of the Universe at large time resembles the scale factor of the inflationary era. One could think of the Universe as having two inflationary eras, one at early times, and the other one after several Hubble times. Instead, if the dark energy is in the form of another accelerating component, the scale factor increases as a power law, as for example in the XCDM model.

The plan of the present paper is the following. In Section 2 we discuss how the dark energy accelerates the Universe and the epoch in which the Universe becomes dark energy dominated; in Section 3 we discuss the flatness of the early and late Universe; in Section 4 we consider a putative late inflationary epoch; and finally in Section 5 we present the conclusions.

2. The accelerating and the dark energy–dominated Universe

In this section we deal with the role of the dark energy in accelerating the Universe. A useful quantity here is the decelerating parameter, $q(a)$, namely

$$q(a) \equiv -\frac{\ddot{a}a}{\dot{a}^2}, \tag{1}$$

(see, e.g., [4]) where the dot stands for time derivatives, and a ($\equiv R/R_0$; scale factor over its present value) is the normalized scale factor. We refer the reader to the appendix, where one finds a series of useful equations used in the present paper.

There are in the literature many different alternatives for the dark energy (see, e.g., [5]), the most usual one is the cosmological constant, Λ. In the present study we restrict our attention to the dark energy in the so called XCDM parametrization, which includes, as a particular case, the cosmological constant.

In the XCDM parametrization the pressure, p_X, is written as

$$p_X = w_X \rho_X, \tag{2}$$

where w_X is a constant and ρ_X is the energy density of the XCDM fluid. The above equation for p_X is also known as the cosmic equation of state.

From the local energy conservation one finds that

$$\rho_X \propto a^{-3(1+w_X)} \tag{3}$$

(see Appendix A for details).

Note that $w_X = -1$ implies that ρ_X is constant. This is nothing but a fluid representation of the cosmological constant. If $w_X < -1$, ρ_X is an increasing function of a. On the other hand, if $w_X > -1$, ρ_X is a decreasing function of a.

Note that the WMAP satellite team (see, e.g, [8]) reported that $w_X < -0.78$, therefore consistent with a cosmological constant.

From the Friedmann equation it is easy to show that $q(a)$ can be written in terms of the density parameters, namely

$$q(a) = \frac{1}{2}\Omega_{\mathrm{m}}(a) + \Omega_{\mathrm{r}}(a) + \frac{1+3w_X}{2}\Omega_X(a); \tag{4}$$

which can be written in terms of the present values of density parameters—$\Omega_{\rm m}$, $\Omega_{\rm r}$ and Ω_X, for matter, radiation and dark energy, respectively—namely

$$q(a) = \frac{\Omega_{\rm m}a + 2\Omega_{\rm r} + (1 + 3w_X)\Omega_X a^{1-3w_X}}{2E(a)}, \qquad (5)$$

where $E(a)$ is found in Appendix A. Note that we have represented the present values of the density parameters simply by "Ω's" and the corresponding subscripts. Hereafter, any density parameter thus represented means its present value, i.e., $\Omega_i \equiv \Omega_i(a = 1)$.

Note that for an accelerating Universe, i.e., $\ddot{a} > 0$, it means that $q(a) < 0$. From the above equation one sees that if

$$w_X < -\frac{1}{3},$$

it may occur that

$$q(a) < 0.$$

From the equations for the evolution of density parameters as a function of the scale factor— found in Appendix A, for example—, one obtains a simple formula either for the redshift, $z_{\rm acc}$, or the scale factor, $a_{\rm acc}$, in which the Universe begins its accelerating epoch, namely

$$a_{\rm acc} = \left(\frac{-1}{1 + 3w_X}\frac{\Omega_{\rm m}}{\Omega_X}\right)^{-1/3w_X}, \qquad (6)$$

and

$$z_{\rm acc} = \left[-(1 + 3w_X)\frac{\Omega_X}{\Omega_{\rm m}}\right]^{-1/3w_X} - 1. \qquad (7)$$

In Fig. 1 some illustrative examples are given on how q varies as a function of the scale factor, a, for a flat Universe model with $\Omega_{\rm m} = 0.30$ and $\Omega_X = 0.70$, for different values of w_X. The lower w_X is, the lower the redshift in which the Universe begins to accelerate. Note that, for $w_X = -1$ the acceleration of the Universe takes place at $z_{\rm acc} = 0.67$.

It is worth mentioning that once the Universe has started accelerating it keeps accelerating forever. The decelerating parameter goes asymptotically to a finite value, namely

$$q(a) \rightarrow \frac{1 + 3w_X}{2} \quad \text{for } w_X < -\frac{1}{3} \quad \text{and} \quad a \gg 1.$$
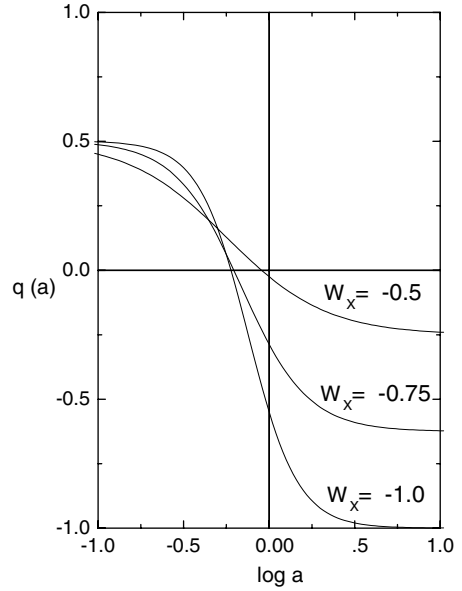

Fig. 1. Evolution of q as a function of a for a flat Universe model with $\Omega_{\rm m} = 0.30$ and $\Omega_X = 0.70$, for different values of w_X.

Now, we show that the epoch in which the Universe becomes dark energy dominated is not the epoch in which it starts accelerating.

The dark energy–dominated Universe begins when $\Omega_X > \Omega_{\rm m}$. From the equations for the evolution of the density parameters one obtains the redshift (the scale factor) $z_{\rm eq}^*(a_{\rm eq}^*)$ of matter–dark energy equality, namely,

$$a_{\rm eq}^* = \left(\frac{\Omega_{\rm m}}{\Omega_X}\right)^{-1/3w_X}, \qquad (8)$$

and

$$z_{\rm eq}^* = \left(\frac{\Omega_X}{\Omega_{\rm m}}\right)^{-1/3w_X} - 1. \qquad (9)$$

Note that there is no the factor of $[-(1 + 3w_X)]$ in the above equations as compared to Eqs. 6 and 7.

The reason why the accelerating and the dark energy–dominated eras begin at different redshifts has to do with the following. The decelerating parameter comes from the Friedman equation, therefore the pressure and the energy density are implicitly taken into account in the calculation of $z_{\rm acc}$ ($a_{\rm acc}$). On the other hand, $z_{\rm eq}^*$ ($a_{\rm eq}^*$) is obtained

by just equating Ω_X with Ω_m, involving therefore only energy densities.

For a flat Universe model with $\Omega_m = 0.30$ and $\Omega_X = 0.70$ for $w_X = -1$ the dark energy–dominated era begins at $z^*_{eq} = 0.33$, which is almost a factor of two lower than z_{acc} for this very model.

As it is well known, coincidentally, the accelerating and the dark energy–dominated era took place quite recently.

3. Flat in the beginning and at the end

For models with $0 < \Omega_m < 1$ and $0 < \Omega_\Lambda < 1$ (where Ω_Λ is the density parameter for the cosmological constant), even for those combinations for which $\Omega_m + \Omega_\Lambda > 1$ the Universe expands forever (see, e.g., [1,4]). It is worth stressing that for these range of values of the density parameters, this effect has to do with the fact that the energy density related to the cosmological constant is constant during the expansion of the Universe which keeps the Universe expanding forever.

Fig. 2 shows some examples on how the total density parameter,

$$\Omega(a) = \Omega_m(a) + \Omega_r(a) + \Omega_X(a), \qquad (10)$$

evolves as a function of the scale factor, for different models, namely: $\Omega = 0.6, 0.8, 1.0, 1.2$ and 1.4, with $\Omega_X = 0.5, 0.6, 0.7, 0.8$ and 0.9, respectively, for $w_X = -1$ (cosmological constant), solid lines, and $w_X = -0.5$, dashed lines. Note that all of them are nearly flat in the beginning of the Universe and in the future, where the dark energy term accounts for this latter evolution. Obviously, most models present in Fig. 2 are merely illustrative, since they are not consistent with the present knowledge one has for the cosmological parameters.

As can be seen in Appendix A, Eq. (10) can be written as follows:

$$\Omega(a) = \frac{\Omega_r + \Omega_m a + \Omega_X a^{1-3w_X}}{E(a)}, \qquad (11)$$

where $E(a)$ is also found in Appendix A.

Note that for small values of a, as is well known, the energy density of the Universe is dominated by the radiation, then

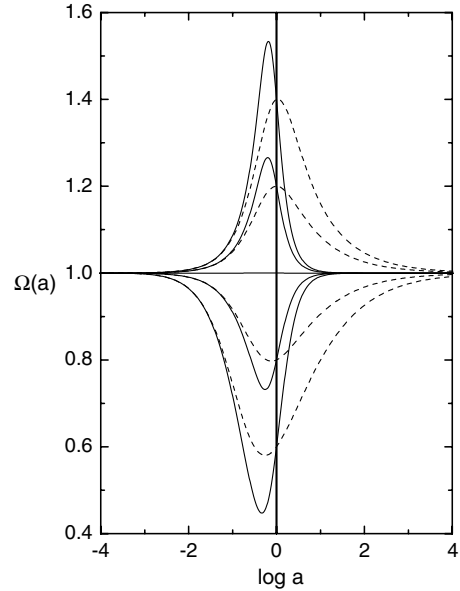$$\Omega(a) \to \Omega_r(a) \to 1 \quad \text{for } a \to 0.$$

Fig. 2. A comparison of the evolution of different Universe models: $\Omega = 0.6, 0.8, 1.0, 1.2$ and 1.4, with $\Omega_X = 0.5, 0.6, 0.7, 0.8$ and 0.9, respectively, for $w_X = -1$ (cosmological constant), solid lines, and $w_X = -0.5$, dashed lines. Note that all of them are nearly flat in the beginning of the Universe and in the future.

Also, the Universe at early time tends to be flat independent of the present values of density parameters.

On the other hand, for large values of a the energy density is dominated by the dark energy, then

$$\Omega(a) \to \Omega_X(a) \to 1 \quad \text{for } w_X < -\frac{1}{3} \quad \text{and} \quad a \gg 1.$$

Moreover, in the same way as that which occurs at early times, the Universe behaves like a flat model, independent of the present values of the density parameters.

In general, any quintessence model, which yields an energy density that evolves with a power law in the scale factor with an exponent greater than -2, has a similar behavior as above.

Therefore, the Universe begins indefinitely flat and will be in the future indefinitely flat, even being either open or closed at intermediate times.

To illustrate how the density parameters of the different components of the Universe evolves, as a function of the scale factor, we present two examples in Figs. 3 and 4.
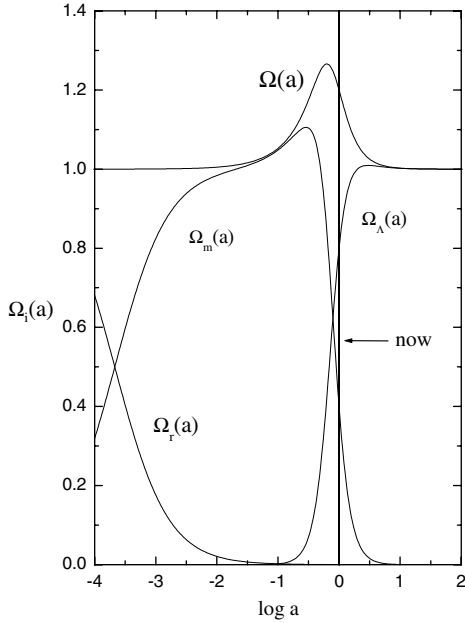
Fig. 3. The evolution of the density parameters as a function of the scale factor for a Universe model with $\Omega = 1.2$, where $\Omega_m = 0.4$ and $\Omega_X = 0.8$. Note that, since we are considering $w_X = -1$ (cosmological constant) the label of the dark energy in the figure is Ω_Λ instead of Ω_X. Note that $\Omega_i(a)$ stands for the different components of the Universe.
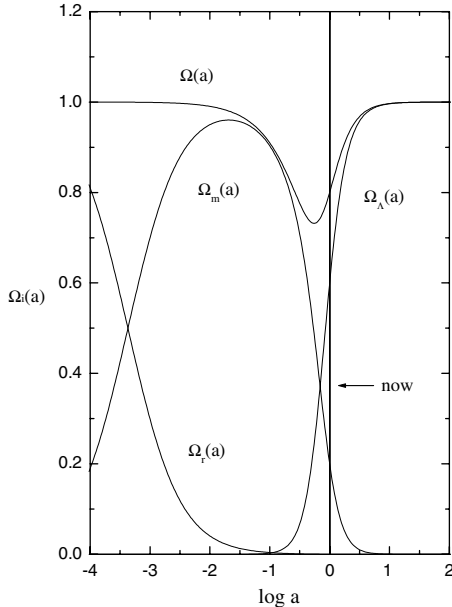

Fig. 4. The same as in Fig. 3 for a Universe model with $\Omega = 0.8$, where $\Omega_m = 0.2$ and $\Omega_\Lambda = 0.6$.

Fig. 3 refers to a closed model with $\Omega = 1.2$, where $\Omega_m = 0.4$ and $\Omega_X = 0.8$. In this figure Ω_i stands for the different components of the Universe. Note that, since we are considering $w_X = -1$ (cosmological constant) the label of the dark energy in the figure is Ω_Λ instead of Ω_X.

Fig. 3 shows, as is already well known, that in the early Universe the dynamics is dominated by the radiation (the radiation–dominated era), after that, at $z \sim 10^4$, up to recent times, the dynamics is dominated by the matter (the matter–dominated era). Moreover, due to the existence of a dark energy component in the Universe, there is the dark energy–dominated era, where the dynamics is dominated by the dark energy. This era begins at a recent time and dominates the dynamics of the Universe indefinitely.

Fig. 4 shows the same as in Fig. 3 now for an open model with a cosmological constant with $\Omega = 0.8$, where $\Omega_m = 0.2$ and $\Omega_\Lambda = 0.6$. The conclusions concerning the evolution of this Universe model are the same as the closed model discussed above.

4. Is there a late inflationary epoch?

Let us now consider how the time evolution of the scale factor is for the XCDM cosmology adopted here, to see, in particular, how the evolution of the Universe is during the dark energy-dominated era.

Note that the time evolution for the scale factor for the radiation-dominated era is not affected by the dark energy. The matter-dominated era is only affected at the end of this era, when the contribution of the dark energy begins to be significant.

The time evolution of the scale factor, for several Hubble times, is completely dominated by the dark energy, and is given by

$$a = \begin{cases} t^{2/[3(1+w_x)]} & \text{for} \quad -1 < w_x \leqslant -1/3 \\ \exp\sqrt{\frac{8\pi G}{3}\rho_x}t & \text{for} \quad w_x = -1 \end{cases}$$

(12)

Note that for $w_X = -1$, i.e., for a cosmological constant, the scale factor evolves exponentially.

This behavior is exactly the same as that which occurs in inflationary cosmology to generate the initial conditions for the big bang.

One could think of the Universe as having two inflationary eras, one at early times and other one at large times, which would cause the Universe to be indefinitely flat at early times and at future times. The difference of this putative late inflation as compared to the early inflation is that the former never stops.

Recall that the de Sitter model is devoid of matter or radiation, there is only the contribution of a cosmological constant, and therefore the Universe expands exponentially, as shown in Eq. (13).

Since for several Hubble times there is no contribution of matter for the dynamics of the expansion of the Universe in the XCDM model considered here, because $\Omega_m \to 0$, this expansion phase of the dark energy-dominated era is the de Sitter one.

On the other hand, in particular for $-1 < w_X \leqslant -1/3$, the Universe expands forever as a power law in the scale factor, and $\Omega \to 1$ for several Hubble times. Again, one could think of this as being an inflationary phase, but now as a power law one.

It is worth noting that the behavior of the XCDM models studied here is given, for several Hubble times, by Eq. (13), irrespective of the Universe at present time being open, flat or closed.

In XCDM accelerating models the Universe never stop accelerating unless the dark energy is such that the value of w_X is not a constant and is less then $-1/3$ in some epoch of the evolution of the Universe.

It is worth mentioning, however, that the dark energy, or the quintessence, could well have a more complex behavior in such a way that it causes an accelerating phase around the present time, and for future times the Universe stops accelerating. This issue is not discussed here and we refer the reader to the literature for details.

5. Conclusions

In this paper we have discussed the role of the dark energy in the evolution of the Universe. We have adopted, in particular, the XCDM parametrization, where the cosmological constant is a particular case of dark energy.

The epochs in which the Universe begun its accelerating phase and when the Universe becomes dark energy–dominated have been studied. In particular, expressions have been derived to calculate the redshifts corresponding to these very epochs, in terms of density parameters of a given Universe model.

Both the early and the late evolutions of the Universe have been addressed. In particular, it has been shown that, for a broad range of density parameters, the Universe expands forever, even though at present time the Universe is closed. The dark energy is responsible for the Universe being asymptotically flat in the future times.

If the dark energy is in the form of a cosmological constant the Universe will expand exponentially at future times, as in the de Sitter model. This resembles the inflationary era of the early Universe. Thus, one can think of as the Universe having two inflationary eras, one at early times and the other one at future times.

For dark energy in which $-1 < w_X \leqslant -1/3$ the Universe expands as a power law, becoming asymptotically flat.

Acknowledgments

Appendix A. Density parameters and related issues

In this appendix we consider a series of equations which are useful in various sections of the present article.

First, the Roberton-Walker metric reads

$$ds^2 = c^2 dt^2 - R^2(t)dl^2, \tag{A.1}$$

where dl^2 depends only on the spatial coordinates, c is the velocity of light, t is the universal time, and $R(t)$ is the scale factor.

A useful quantity is the normalized scale factor, $a(t)$, which is related to the scale factor as follows:

$$a(t) = \frac{R(t)}{R_0}, \tag{A.2}$$

where R_0 is the present value of the scale factor, which can be taken to be equal to unity.

Concerning the cosmological parameters it is implicit in many textbooks and papers that one is referring to their present values. It is well known, however, that the density parameters evolve with the expansion of the Universe.

Whenever the density parameters appear in the present paper without any functional dependence, it means that we are referring to their present values.

Let us look at how the density parameters evolve in an expanding Universe. Recall that the density parameter is defined as follows:

$$\Omega_i(a) \equiv \frac{\rho_i(a)}{\rho_c(a)}, \tag{A.3}$$

where $\rho_i(a)$ is the energy density of ith component i.e., baryons, photons, dark matter, and dark energy, for example as a function of the scale factor a, and $\rho_c(a)$ is the critical density as a function of a.

Recalling that the Friedmann equation reads

$$\dot{R}^2 - \frac{8\pi G}{3}\rho R^2 = -kc^2, \tag{A.4}$$

where k is either -1, 0 or $+1$, for open, flat and closed Universes, respectively. Adopting $k = 0$ one can obtain from the above equation an expression for the density, which is called critical density, i.e., the density of a flat Universe.

Thus, the critical density reads

$$\rho_c(a) = \frac{3H^2}{8\pi G}, \tag{A.5}$$

and

$$H = \frac{\dot{R}}{R} \tag{A.6}$$

is the Hubble parameter, and the "dot" stands for time derivatives.

It is worth mentioning that instead of following the time evolution of the density parameters we follow, throughout the paper, their evolution

either as function of the scale factor (a or R) or as function of the redshift, z. These very quantities are related to each other as follows:

$$a = \frac{R}{R_0} = \frac{1}{1+z}.$$

The energy content of the Universe is divided into pressureless matter, in the form of baryonic matter and non baryonic weakly interacting cold dark matter, radiation, and dark energy.

The matter and the radiation energy densities depend on the scale factor as follows

$$\rho_m \propto R^{-3}, \tag{A.7}$$

$$\rho_r \propto R^{-4}, \tag{A.8}$$

For the dark energy, recall that in the XCDM parametrization, the pressure is written

$$p_X = w_X \rho_X. \tag{A.9}$$

From the local energy conservation, namely

$$\dot{\rho} = -3H(t)(\rho + p), \tag{A.10}$$

it follows that

$$\dot{\rho}_X = -3\frac{\dot{a}}{a}\rho_X(1 + w_X), \tag{A.11}$$

thus

$$\rho_X \propto a^{-3(1+w_X)} \tag{A.12}$$

Now, from the Friedmann equation one obtains

$$\frac{kc^2}{H^2 R^2} = \Omega_m(a) + \Omega_r(a) + \Omega_X(a) - 1 \tag{A.13}$$

Defining the curvature density parameter as

$$\Omega_k(a) \equiv -\frac{kc^2}{H^2 R^2}, \tag{A.14}$$

it follows that

$$\Omega_m(a) + \Omega_r(a) + \Omega_X(a) + \Omega_k(a) = 1; \tag{A.15}$$

this means that if $\Omega_k(a)$ is added to the other density parameters one gets a unity value.

In the present paper, Ω without a subscript denotes the total density parameter without the curvature term, namely

$$\Omega(a) = \Omega_m(a) + \Omega_r(a) + \Omega_X(a) \tag{A.16}$$

Now, we present how the density parameters evolve as function of a. It is easy to show that

$$\frac{8\pi G\rho}{3} = H_0^2(\Omega_X a^{-3(1+w_X)} + \Omega_{\mathrm{m}}a^{-3} + \Omega_{\mathrm{r}}a^{-4}).$$

$$(A.17)$$

Using Eq. (A.13) one obtains

$$H^2(a) = H_0^2[\Omega_X a^{-3(1+w_X)} + \Omega_k a^{-2} \\ + \Omega_{\mathrm{m}}a^{-3} + \Omega_{\mathrm{r}}a^{-4}].$$

$$(A.18)$$

Finally, using the definition of density parameter for each component of the Universe one obtains

$$\Omega_{\mathrm{r}}(a) = \frac{\Omega_{\mathrm{r}}}{E(a)},$$

$$(A.19)$$

$$\Omega_{\mathrm{m}}(a) = \frac{\Omega_{\mathrm{m}}}{E(a)}a,$$

$$(A.20)$$

$$\Omega_k(a) = \frac{\Omega_k}{E(a)}a^2,$$

$$(A.21)$$

$$\Omega_X(a) = \frac{\Omega_X}{E(a)}a^{1-3w_X},$$

$$(A.22)$$

$$\Omega(a) = \frac{\Omega_{\mathrm{r}} + \Omega_{\mathrm{m}}a + \Omega_X a^{1-3w_X}}{E(a)};$$

$$(A.23)$$

where

$$E(a) = \Omega_{\mathrm{r}} + \Omega_{\mathrm{m}}a + \Omega_k a^2 + \Omega_X a^{1-3w_X}.$$

$$(A.24)$$

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